BST 675 — Fall 2010 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Wednesday 22 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[20] 1. Consider the sample space $S := \{1, 2, 3, 4, 5, 6, 7, 8\}$. Suppose that all elements in S are equally likely. Put $A := \{1, 2, 3, 4\}$ and $B := \{1, 3, 5, 7\}$.

[10] a. Show that A and B are independent.

[10] b. Exhibit an event C such that

$$P(A \cap C) = P(A)P(C) \quad \text{and} \quad P(B \cap C) = P(B)P(C) \quad \text{but} \quad P(A \cap B \cap C) \neq P(A)P(B)P(C).$$

Hence, three events are not automatically guaranteed to be independent just because any two of them are independent.

Remark: The intuition here is that, while knowing A has occurred may not be informative about whether C will occur and knowing B has occurred may not be informative about whether C will occur, knowing A and B both have occurred may be very informative about whether C will occur.

[40] 2. Suppose that eight people who wish to quit smoking enroll in a smoking cessation support program and that we are able to find out who has successfully quit smoking six months later. Letting Q be shorthand for "Quit" and N be shorthand for "Did Not Quit", we can formulate a sample space S consisting of elements such as $QQNNNQQN$ and $NNQNQNQQ$. Suppose that all elements of S are equally likely.

Remark: Implicit in this supposition is the belief that, for any person enrolling in the program, there is a 50% chance of success. Also implicit in this supposition is the belief that one participant's success does not make any more or less likely another participant's success.

[10] a. What is the probability that all of the eight participants successfully quit smoking?

[10] b. What is the probability that exactly seven of the eight participants successfully quit smoking?

[10] c. What is the probability that exactly six of the eight participants successfully quit smoking?

[10] d. Use one or more axioms of probability to explain why the probability that at least six of the eight participants successfully quit smoking equals the sum of the answers to parts a through c.

Remark: If we let X denote the number of participants who successfully quit smoking, then what we have here is $P(6 \leq X \leq 8) = P(X = 6) + P(X = 7) + P(X = 8)$. Do you remember taking such additions for granted in your introductory statistical methods course? Now you can see why they are correct.

[20] 3. I am dealt seven cards from a well shuffled 52 card deck. What is the probability that I get two pairs and a triple?

Caution: The number of ways to select the denominations for the two pairs is not 13×12 because, for example, a pair of 9's and a pair of 10's is the same as a pair of 10's and a pair of 9's.

[20] 4. In a "Pick 4" lottery game, you select a four digit number. Ten balls marked with the digits 0 through 9 are placed in a vat, from which four selections are made with replacement. You win a prize if any reordering of the four selections matches your four digit number. For example, if you have selected the four digit number 1122, then you win a prize if the four selections are 1122, 1212, 1221, 2112, 2121, or 2211. Can you propose a good strategy for playing this lottery game?