BST 675 — Fall 2010 — Dr. Charnigo

Solutions to Written Assignment 2

1a. We have $P(A) = 4/8$ since A contains four out of eight elements and likewise $P(B) = 4/8$ and $P(A \cap B) = 2/8$. Since $P(A)P(B) = 1/2 \times 1/2 = 1/4 = P(A \cap B)$, we conclude that A and B are independent.

1b. Put $C := \{1, 3, 6, 8\}$. Then $A \cap C = B \cap C = A \cap B \cap C = \{1, 3\}$ so that $P(A \cap C) = 1/4 = 1/2 \times 1/2 = P(A)P(C)$, $P(B \cap C) = 1/4 = 1/2 \times 1/2 = P(B)P(C)$, and $P(A \cap B \cap C) = 1/4 \neq 1/2 \times 1/2 \times 1/2 = P(A)P(B)P(C)$.

2a. Let A_8 denote the event that all eight participants successfully quit smoking. We have $card(A_8) = 1$ (there is only one string with all Q's) and $card(S) = 2^8$, so $P(A_8) = 1/2^8 = 1/256$.

2b. Let A_7 denote the event that exactly seven participants successfully quit smoking. We have $card(A_7) = 8$ (there are eight strings with exactly one N because there are eight choices for where to place the N) and $card(S) = 2^8$, so $P(A_7) = 8/2^8 = 1/32$.

2c. Let A_6 denote the event that exactly six participants successfully quit smoking. We have $card(A_6) = 8 \times 7/2 = 28$ (there are 28 strings with exactly two N's because there are eight choices for where to place one of the N's, seven choices for where to place the other N, and each of these 56 choices is redundant of one other choice in the sense of yielding an identical string) and $card(S) = 2^8$, so $P(A_6) = 28/2^8 = 7/64$.

2d. Since A_6, A_7, and A_8 are mutually exclusive, the third axiom of probability tells us that $P(A_6 \cup A_7 \cup A_8) = P(A_6) + P(A_7) + P(A_8)$. Of course, $A_6 \cup A_7 \cup A_8$ is the event that at least six participants successfully quit smoking.

3. The number of ways to choose seven cards from a well shuffled 52 card deck is $\binom{52}{7} = 133784560$, and we may assume that all are equally likely. The number of ways to choose the denominations of the two pairs is $\binom{13}{2}$, and then the number of ways to choose the denomination of the triple is 11. The number of ways to choose the suits for each pair is $\binom{4}{2}$. The number of ways to choose the suits for the triple is $\binom{4}{3}$. So, the number of ways to choose seven cards with two pairs and a triple is $\binom{13}{2} 11 \binom{4}{2}^2 \binom{4}{3} = 123552$. Hence, the requested probability is $123552/133784560 = 0.00092$.

4. The best strategy is to choose a number whose four digits are distinct, for then there are $24 = 4 \times 3 \times 2 \times 1$ ways to win. If you choose a number with one digit repeated twice, there are $12 = 4 \times 3 \times \binom{2}{2}$ ways to win. If you choose a number with one digit repeated three times, there are $4 = 4 \times \binom{3}{3}$ ways to win. If you choose a number with two digits repeated twice, there are $6 = \binom{4}{2} \times \binom{2}{2}$ ways to win. If you choose a number with one digit repeated four times, there is only 1 way to win.