

# BST 675 — Fall 2010 — Dr. Charnigo

## Written Assignment 3

Written Assignment 3 is due on Wednesday 20 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[80] 1. Suppose that  $X$  has probability density function  $Cx^2(1-x)1_{\{0 < x < 1\}}$  for some constant  $C$ .

[10] a. What is  $C$ ?

[10] b. What is the cumulative distribution function of  $X$ ?

[10] c. Let  $p$  be a real number. Evaluate  $E[X^p]$ . *Hint:* Consider separately the cases  $p > -3$  and  $p \leq -3$ .

[10] d. What is the cumulative distribution function of  $Y := -\log X$ ?

[10] e. What is the probability density function of  $Y$ ?

[10] f. What is the survival function of  $Y$ ?

[10] g. What is the hazard function of  $Y$ ?

[10] h. What is the expected value of  $Y$ ?

[20] 2. Suppose that  $X$  has probability density function  $(2\pi\sigma^2)^{-1/2} \exp[-(x-\mu)^2/(2\sigma^2)]$ , where  $\mu \in (-\infty, \infty)$  and  $\sigma \in (0, \infty)$  are constants. Put  $Z := (X-\mu)/\sigma$ , so that  $Z$  has probability density function  $(2\pi)^{-1/2} \exp[-z^2/2]$ .

[10] a. Show that the moment generating function  $M_Z(t) = \exp[t^2/2]$ . *Hint:* First prove that  $\exp[tz] \exp[-z^2/2] = \exp[-(t-z)^2/2] \exp[t^2/2]$ .

[10] b. Show that the moment generating function  $M_X(t) = \exp[t\mu]M_Z(\sigma t)$ . Conclude that  $M_X(t) = \exp[t\mu + t^2\sigma^2/2]$ .