BST 675 — Fall 2010 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Wednesday 20 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[80] 1. Suppose that X has probability density function $Cx^2(1-x)1_{\{0<x<1\}}$ for some constant C.

[10] a. What is C?

[10] b. What is the cumulative distribution function of X?

[10] c. Let p be a real number. Evaluate $E[X^p]$. *Hint*: Consider separately the cases $p > -3$ and $p \leq -3$.

[10] d. What is the cumulative distribution function of $Y := -\log X$?

[10] e. What is the probability density function of Y?

[10] f. What is the survival function of Y?

[10] g. What is the hazard function of Y?

[10] h. What is the expected value of Y?

[20] 2. Suppose that X has probability density function $(2\pi\sigma^2)^{-1/2}\exp[-(x-\mu)^2/(2\sigma^2)]$, where $\mu \in (-\infty, \infty)$ and $\sigma \in (0, \infty)$ are constants. Put $Z := (X-\mu)/\sigma$, so that Z has probability density function $(2\pi)^{-1/2}\exp[-z^2/2]$.

[10] a. Show that the moment generating function $M_Z(t) = \exp[t^2/2]$. *Hint*: First prove that $\exp[tz]\exp[-z^2/2] = \exp[-(t-z)^2/2]\exp[t^2/2]$.

[10] b. Show that the moment generating function $M_X(t) = \exp[t\mu]M_Z(\sigma t)$. Conclude that $M_X(t) = \exp[t\mu + t^2\sigma^2/2]$.