BST 675 — Fall 2011 — Dr. Charnigo

In-Class Assessment

This assessment is a strictly individual activity. Textbooks, notes, calculators, computers, and technology with Internet access are prohibited. Record what you want graded in the blue book.

[20] 1. Let S denote a sample space, and let $A_1, A_2, \ldots$ denote mutually exclusive events within that sample space.

[10] a. State the three axioms of probability.

[10] b. A student draws seven cards without replacement from a well-shuffled standard 52-card deck. What is the probability that the student draws at least six hearts ?

[20] 2. Let X have cumulative distribution function $F(x) := (1 - \exp[-3x])1_{x>0}$.

[10] a. Find the probability density function of X. What is the name of the commonly encountered parametric family to which this probability density function belongs ?

[10] b. Find the probability density function of $Y := -(1/3)\log X$.

[30] 3. Let X and Y be independent binomial random variables based on 2 trials with respective success probabilities 0.5 and 0.5. Let U and V be independent binomial random variables based on 2 trials with respective success probabilities 0.3 and 0.7.

[10] a. State the definition of a moment generating function for a generic random variable W.

[10] b. Recalling that the moment generating function for a binomial random variable based on n trials with success probability $p \in [0,1]$ is $(pe^t + 1 - p)^n$, prove that $X + Y$ is a binomial random variable.

[10] c. Calculate $E[U + V]$ and $Var[U + V]$. Show that there does not exist $p \in [0,1]$ such that, simultaneously, $4p = E[U + V]$ and $4p(1 - p) = Var[U + V]$. Conclude that $U + V$ is not a binomial random variable.

[30] 4. Let X and Y have joint probability mass function

$$f_{X,Y}(x, y) := e^{-\lambda}e^{-\mu}\lambda^x\mu^{y-x}1_{x\leq y}/\{x!(y - x)!\}$$

for nonnegative integers x and y, where λ and μ are positive real numbers.

[10] a. Explain how you know, even before calculating the marginal probability mass functions of X and Y, that X and Y cannot be independent.

[10] b. Explain why $\sum_{x=0}^{y} \left(\frac{\lambda}{\lambda+\mu}\right)^x \left(\frac{\mu}{\lambda+\mu}\right)^{y-x} \frac{y!}{x!(y-x)!} = 1$. Use this fact to evaluate $\sum_{x=0}^{y} \frac{\lambda^x\mu^{y-x}}{x!(y-x)!}$ and thereby determine the marginal probability mass function of Y.

[10] c. Explain why $\sum_{z=0}^{\infty} e^{-\mu}\mu^z/z! = 1$. Use this fact to evaluate $\sum_{y=x}^{\infty} e^{-\mu}\mu^{y-x}/(y - x)!$ and thereby determine the marginal probability mass function of X.