BST 675 — Fall 2011 — Dr. Charnigo

Final Examination

This non-collaborative take-home final examination is due at 5:30 p.m. on Tuesday 13 December.

[50] 1. Let X and Y be independent binomial random variables, based on n_1 and n_2 trials respectively (positive integers, not necessarily equal) with common success probability $p \in (0, 1)$.

[10] a. For two independent but otherwise generic random variables S and T, show that $M_{S-T}(t) = M_S(t)M_T(-t)$ whenever all of these quantities exist finitely.

[10] b. Evaluate $M_{X-Y}(t)$. (You may quote without proof formulas of moment generating functions for binomial variables.)

[10] c. Identify a random variable W, possibly degenerate, such that $M_W(t) = \exp[n_2 t]$.

[10] d. Evaluate $M_{X-Y+W}(t)$. Supposing that $p = 1/2$, what is the probability mass function of $X - Y + W$?

[10] e. Supposing that $p = 1/2$, what is the probability mass function of $X - Y$?

[50] 2. Let X and Y be independent chi-square random variables on two degrees of freedom. Let $U := X - Y$ and $V := Y$.

[10] a. Use part a of exercise 1 to evaluate $M_U(t)$. Do an Internet search to find a formula of the moment generating function for a random variable with the "Laplace distribution". Thereby determine the marginal probability density function of U. (You may quote without proof formulas of moment generating functions for chi-square variables.)

[10] b. Find the joint probability density function of U and V.

[10] c. Confirm your answer to part a of exercise 2 by integrating the joint probability density function obtained in part b of exercise 2. (Either consider the cases $u \geq 0$ and $u < 0$ separately or consider just the case $u \geq 0$ and then argue that the marginal probability density function of U must be symmetric about 0.)

[10] d. Find $Var[U]$. (You can appeal to symmetry to obtain an integral from 0 to ∞, which will be amenable to integration by parts or to application of the kernel method. Alternatively, you can employ the moment generating function.)

[10] e. We see that adding the non-degenerate random variable U to a chi-square random variable on two degrees of freedom (Y) produces another chi-square random variable on two degrees of freedom (X). In particular, we have $Var[X] = Var[Y]$. Explain why we do not have $Var[X] = Var[U + Y] = Var[U] + Var[Y] > Var[Y]$.