BST 675 — Fall 2011 — Dr. Charnigo

Written Assignment 1

Written Assignment 1 is due on Tuesday 20 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[10] 1. Give a careful proof of the second distributive law, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

[10] 2. Prove result 9 that $P(A) \leq P(B)$ whenever $A \subset B$, without using any subsequent result. (Thus, you are limited to axioms 1, 2, 3 and results 4, 5, 6, 7, 8.)

[10] 3. I will be dealt 7 cards from a well-shuffled standard 52-card deck. What is the probability that I will receive one triple and two pairs?

[10] 4. Consider two lottery games. Game A entails selecting 3 balls without replacement from a vat containing 30 balls numbered from 1 to 30; you win if you guess at least two of the numbers correctly. Game B entails selecting 6 balls without replacement from a vat containing 60 balls numbered 1 to 60; you win if you guess at least four of the numbers correctly. With which game do you have a better chance of winning?

[10] 5. Provide an example in which events A, B, C are not independent even though A, B are independent, A, C are independent, and B, C are independent.

[10] 6. Sometimes people use a Bonferroni correction in post-hoc testing after one-way analysis of variance. This entails setting the significance level of each post-hoc test to α/m, where m is the total number of post-hoc tests to be performed. The idea is that the probability of incorrectly rejecting at least one true null hypothesis will then be no more than α.

For $i \in \{1, 2, \ldots, m\}$, let A_i denote the event that the null hypothesis in post-hoc test i is rejected and B_i denote the event that the null hypothesis in post-hoc test i is true. Justify the Bonferroni correction by proving that $P(\cup_{i=1}^{m} \{A_i \cap B_i\}) \leq \alpha$.

[20] 7. Suppose that 0.8% (8/10 of 1%, not 80%) of people in the U.S. have HIV and that a diagnostic test is available such that: (i) P% of people who really have HIV test positive; and, (ii) Q% of people who really do not have HIV test negative, where P and Q are some numbers between 0 and 100. Indicate whether each of the following statements is true or false; justify your answers.

a. If $P = 100$, then there are no false positive test results.
b. If $P = 100$, then there are no false negative test results.
c. If $Q = 100$, then there are no false positive test results.
d. If $Q = 100$, then there are no false negative test results.

[20] 8. In each of the following cases, for what value(s) of C (if any) is $F(x)$ a valid cumulative distribution function for a continuous random variable? a discrete random variable? a random variable that is neither discrete nor continuous?

a. $F(x) := \sin(Cx)1_{x\in[0,1]} + 1_{x>1}$
b. $F(x) := \cos(Cx)1_{x\in[0,1]} + 1_{x>1}$
c. $F(x) := \sin(Cx)1_{x\in(0,1)} + 1_{x\geq 1}$
d. $F(x) := \cos(Cx)1_{x\in(0,1)} + 1_{x\geq 1}$