BST 675 — Fall 2011 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Tuesday 01 November at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. Let X_n have the chi-square distribution on n degrees of freedom for $n \in \{1, 2, \ldots\}$. In what follows, you may quote without proof the facts that X_n has mean n, variance $2n$, and moment generating function $M_{X_n}(t) := (1 - 2t)^{-n/2}$ for $t < 1/2$.

[10] a. Put $Z_n := (2n)^{-1/2}(X_n - n)$. What is $M_{Z_n}(t)$?

[10] b. What is $\lim_{n\to\infty} M_{Z_n}(t)$?

[10] c. Use result b to ascertain $\lim_{n\to\infty} P(X_n \le z(2n)^{1/2} + n)$ for any fixed real number z. (You may express your answer in integral form.)

[10] d. What does result c tell you about the shape of the chi-square distribution on n degrees of freedom when n is large?

[10] e. Employ conclusion d to obtain an approximation to $P(180 \le X_{200} \le 220)$. Then use software, such as R or SAS, to calculate $P(180 \le X_{200} \le 220)$ to four decimal places. Repeat the preceding steps for $P(0 \le X_2 \le 4)$. Thereby observe that conclusion d is not applicable when n is small.

[20] 2. Let $F(x, y) := (xy)^2 \sin[(xy)^2]$.

[10] a. Find $\frac{\partial}{\partial x} F(x, y)$ and $\frac{\partial}{\partial y} F(x, y)$.

[10] b. Find $\frac{\partial^2}{\partial x^2} F(x, y)$, $\frac{\partial^2}{\partial x \partial y} F(x, y)$, and $\frac{\partial^2}{\partial y^2} F(x, y)$.

[10] 3. Let $f(x, y) := \exp(-x^2)$ and $R := \{(x, y) : 0 \le x \le y \le 1\}$. Find $\int\int_R f(x, y)\, dx\, dy$.

[20] 4. Let $\mathbf{b} := (-1, 4)^T$, $\mathbf{x} := (x_1, x_2)^T$, and

$$\mathbf{A} := \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix},$$

where x_1 and x_2 are real numbers.

[10] a. Evaluate $\mathbf{Ab}$.

[10] b. Show that $\mathbf{x}^T\mathbf{Ax}$ has the representation $c_1 x_1^2 + c_2 x_2^2 + c_3(x_1 + x_2)^2$, where c_1, c_2, and c_3 are nonnegative constants that you will calculate. Then use this representation to prove that $\mathbf{A}$ is positive definite.