BST 676 — Spring 2010 — Dr. Charnigo

Midterm Examination

The Midterm Examination, a strictly individual activity, may be submitted any time between 12 Noon on Monday 08 March and 5 p.m. on Wednesday 10 March.

Suppose that $X_1, X_2, \ldots, X_n$ are independent and identically distributed Poisson random variables with parameter $\theta \in \Theta := (0, \infty)$. Following are some potentially useful facts that you may quote without proof:

Fact #1. The mean and variance of the Poisson distribution with parameter θ are both equal to θ.

Fact #2. The sum of n independent Poisson random variables with parameter θ is itself a Poisson random variable with parameter $n\theta$.

Fact #3. For any real number λ we have $\sum_{t=0}^{\infty} \lambda^t / t! = \exp[\lambda]$.

[15] 1. Evaluate the Cramer-Rao lower bound for unbiased estimation of θ.

[10] 2. Find the best unbiased estimator of θ. For subsequent items, let the best unbiased estimator be denoted $\hat{\theta}$.

[15] 3. Prove that $\hat{\theta}$ is consistent for θ.

[10] 4. Show that $\exp[-\hat{\theta}]$ is not unbiased for $\exp[-\theta]$ and hence cannot be the best unbiased estimator of $\exp[-\theta]$.

[15] 5. Describe the large sample behavior of $n^{1/2}(\exp[-\hat{\theta}] - \exp[-\theta])$.

[10] 6. Prove that $\exp[-\hat{\theta}]$ is consistent for $\exp[-\theta]$.

[15] 7. Show that $[(n-1)/n]^{\sum_{i=1}^{n} X_i}$ is unbiased for $\exp[-\theta]$ and find its variance.

[10] 8. Prove that $[(n-1)/n]^{\sum_{i=1}^{n} X_i}$ is consistent for $\exp[-\theta]$. [In fact, this is the best unbiased estimator, although one needs the machinery of complete statistics to prove it.]