BST 676 — Spring 2010 — Dr. Charnigo

Mock Comprehensive Examination

This examination is a strictly individual activity. Textbooks, notes, calculators, computers, and technology with Internet access are prohibited. Record that which you want graded in the blue book.

[20] 1. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta) = a(x)b(\theta)\exp[c(x)d(\theta)]$ for $\theta \in \Theta \subset \mathbb{R}$.

[10] a. State the general definition of Fisher information for n observations.

[10] b. State the general definition of the Cramer-Rao lower bound for unbiased estimation of θ.

[30] 2. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta) = \theta^x(1-\theta)^{1-x}$ for $x \in \{0,1\}$ and $\theta \in (0,1)$.

[10] a. Find the best unbiased estimator of θ.

[10] b. Show that the estimator in part a is consistent for θ.

[10] c. Exhibit a consistent estimator of $Var_\theta[X_1]$. (The estimator need not be unbiased.)

[30] 3. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\theta, 1)$ for $\theta \in \mathbb{R}$.

[10] a. Find the maximum likelihood estimator of θ. (You may assume that you do, in fact, obtain the global maximizer of the likelihood by setting the first derivative of the log likelihood equal to zero.)

[10] b. Write out the likelihood ratio test statistic for $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Simplify the result to express it as a function of $(\bar{X} - \theta_0)^2$.

[10] c. For what values of the likelihood ratio test statistic in part b should you reject H_0 if you desire a test with significance level α?

[20] 4. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} Pois(\theta)$ for $\theta \in (0, \infty)$.

[10] a. The Wald test statistic for $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ is $(\bar{X} - \theta_0)/\sqrt{\bar{X}/n}$. Derive an approximate $100(1-\alpha)\%$ Wald interval for θ.

[10] b. The score test statistic for $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ is $(\bar{X} - \theta_0)/\sqrt{\theta_0/n}$. Derive an approximate $100(1-\alpha)\%$ score interval for θ. (The solutions of $ay^2 + by + c = 0$ are $y = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.)