BST 676 — Spring 2010 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Wednesday 17 February at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[25] 1. This exercise will illustrate that the method of moments and maximum likelihood may not produce the same estimates.

[10] a. Find the method of moments estimate of θ for Example #5 in Unit II.

[15] b. Find the maximum likelihood estimate of θ for Example #3 in Unit II. [Let ζ_1 and ζ_2 denote the components of the two-dimensional vector ζ. Differentiate the log likelihood in ζ_1 and set the result equal to zero. Then differentiate the log likelihood in ζ_2 and set the result equal to zero. The solutions to these two equations will be the components of $\widehat{\theta}$.]

[25] 2. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with probability density function $f(x;\theta) := 3\theta^3/x^4$ for $x \in [\theta, \infty)$ and $\theta \in \Theta := (0, \infty)$.

[10] a. Find the method of moments estimate of θ.

[15] b. Find the maximum likelihood estimate of θ.

[25] 3. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with probability mass function $f(x;\theta) := \theta^x(1-\theta)^{1-x}$ for $x \in \{0, 1\}$ and $\theta \in \Theta := [0, 1]$. [If $\theta = 0$ or $\theta = 1$, then the distribution is degenerate.]

[10] a. Find the maximum likelihood estimate of θ.

[15] b. Impose prior distribution $p(\theta) \propto \theta^a(1-\theta)^b$ for specified nonnegative constants a and b. Find the posterior mean and posterior mode estimates of θ.

[25] 4. In their 2008 *Journal of Applied Statistics* paper "Omnibus testing and gene filtration in microarray data analysis", Dai and Charnigo define a test statistic D_n such that, when the null hypothesis is true, $nD_n \xrightarrow{L} \tau_1 Y_1 + \tau_2 Y_2$, where τ_1, τ_2 are known positive constants and Y_1, Y_2 are independent chi-square random variables on one degree of freedom. If nD_n is larger than the 0.95 quantile of $\tau_1 Y_1 + \tau_2 Y_2$, we reject the null hypothesis at approximate significance level 0.05.

[10] a. Suppose that we wish to approximate $\tau_1 Y_1 + \tau_2 Y_2$ by aY, where a is a nonnegative constant and Y is a chi-square random variable on ν degrees of freedom. Propose choices for a and ν.

[15] b. Suppose further that $\tau_1 = 1/(4\sqrt{\pi})$ and $\tau_2 = 3/(16\sqrt{\pi})$. Let $\chi^2_{\nu,0.95}$ denote the 0.95 quantile of the chi-square distribution on ν degrees of freedom, which you can extract from R or other software of your choice. Perform a simulation experiment with R or other software of your choice to estimate $P(\tau_1 Y_1 + \tau_2 Y_2 > a\chi^2_{\nu,0.95})$. [Simulate 10000 realizations of $\tau_1 Y_1 + \tau_2 Y_2$ and record how many of them exceed $a\chi^2_{\nu,0.95}$.] Comment on the implications of using aY as an approximation to $\tau_1 Y_1 + \tau_2 Y_2$ in the context of the above hypothesis test.