BST 676 — Spring 2010 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Wednesday 03 March at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

Suppose that $X_1, X_2, \ldots, X_n \stackrel{iid}{\sim} f(x;\theta) := \theta x^{\theta-1}$ for $x \in (0,1)$ and $n \geq 3$, where $\theta \in \Theta := (0,\infty)$.

[10] 1. Evaluate $E_\theta[-\log X_1]$ and $E_\theta[(-\log X_1)^2]$. [Make the substitution $w := -\log x$ and $dw := -dx/x$. Then integrate by parts.]

[10] 2. Evaluate the Cramer-Rao lower bound for unbiased estimation of $\tau := 1/\theta$.

[10] 3. Find the best unbiased estimator of τ. For subsequent items, let the best unbiased estimator be denoted $\hat{\tau}$.

[10] 4. Find $a \in (0,\infty)$, possibly depending on n, for which the mean square error of $a\hat{\tau}$ is minimal.

[10] 5. Find $a \in (0,\infty)$, possibly depending on n, for which $a\hat{\tau}$ minimizes criterion (13) from Unit III.

[10] 6. Prove that $\hat{\tau}$ is consistent for τ.

[10] 7. Show that $1/\hat{\tau}$ is not unbiased for θ and therefore cannot be the best unbiased estimator of θ.

[10] 8. Prove that $1/\hat{\tau}$ is consistent for θ.

[10] 9. Find $a \in (0,\infty)$, possibly depending on n, so that $a/\hat{\tau}$ is unbiased for θ. [Deriving the probability density function of $-\log X_1$ may be a helpful first step.]

[10] 10. Prove that the unbiased estimator from item 9 is consistent for θ.