BST 676 — Spring 2010 — Dr. Charnigo

Written Assignment 4

Written Assignment 4 is due on Wednesday 31 March at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[20] 1. Suppose that you obtain the following adverse events data.

	Medication	Placebo	Row Total
Adverse Event	6	1	7
No Adverse Event	987	996	1983
Column Total	993	997	1990

Let p_1 denote the rate of adverse events on medication and p_2 the rate of adverse events on placebo. Use Fisher's exact test to decide whether to reject $H_0 : p_1 = p_2$ in favor of $H_1 : p_1 \neq p_2$ at significance level 0.05. First obtain an answer using pencil, paper, and hand calculator. Then compare your answer to that acquired from a statistical software package.

[60] 2. Let $x_1, x_2, \ldots, x_n$ be known constants, not all equal. Consider the linear regression model $Y_i = \alpha + \beta x_i + \epsilon_i$ for $i \in \{1, 2, \ldots, n\}$, where $\epsilon_i \overset{iid}{\sim} N(0, 1)$ and α, β are unknown real constants.

[10] a. Write out the likelihood function.

[10] b. Argue that maximizing the likelihood function minimizes $\sum_{i=1}^{n}(Y_i - \zeta_1 - \zeta_2 x_i)^2$. Then derive the maximum likelihood estimators of α and β. (You can check your answer against Equation 11.3 in Rosner's Fundamentals of Biostatistics, Sixth Edition.)

[10] c. Consider testing $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$. Find the maximum likelihood estimator of α under H_0 by minimizing $\sum_{i=1}^{n}(Y_i - \zeta_1 - \zeta_2 x_i)^2$ in ζ_1 with ζ_2 constrained to equal 0.

[10] d. Calculate the likelihood ratio test statistic for $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$. Show that negative twice the logarithm of the likelihood ratio test statistic equals the regression sum of squares $\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$, where $\hat{Y}_i := \hat{\alpha} + \hat{\beta} x_i$ for $i \in \{1, 2, \ldots, n\}$.

[10] e. Prove that $\hat{\beta}$ follows a normal distribution with expected value β and variance $1/\sum_{i=1}^{n}(x_i - \bar{x})^2$. Some useful facts for this proof are that $\sum_{i=1}^{n}(x_i - \bar{x})C = 0$ and

$$\sum_{i=1}^{n}(x_i - \bar{x})Dx_i = \sum_{i=1}^{n}(x_i - \bar{x})Dx_i - \bar{x}\sum_{i=1}^{n}(x_i - \bar{x})D = \sum_{i=1}^{n}(x_i - \bar{x})D(x_i - \bar{x}) = D\sum_{i=1}^{n}(x_i - \bar{x})^2$$

for any constants C and D.

[10] f. Show that the regression sum of squares equals $\hat{\beta}^2 \sum_{i=1}^{n}(x_i - \bar{x})^2$. Hence, if H_0 is true, the regression sum of squares follows a chi-square distribution on one degree of freedom. What, then, is an appropriate critical value for the likelihood ratio test statistic?

[20] 3. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed as Poisson random variables with unknown parameter $\theta \in (0, \infty)$. Let θ_0 be a known fixed element of $(0, \infty)$. (In what follows, you may quote without proof results that you established in the Midterm Examination.)

[10] a. Derive a Wald test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[10] b. Derive a score test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.