BST 676 — Spring 2010 — Dr. Charnigo

Written Assignment 5

Written Assignment 5 is due on Wednesday 14 April at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. We continue with the scenario from exercise 2 of Written Assignment 4.

[10] a. Suppose that $x_i = (2i-1)/n - 1$ for $i \in \{1, \ldots, n\}$. Noting that $\bar{x} = 0$ by symmetry, evaluate $\sum_{i=1}^{n}(x_i - \bar{x})^2$. (The summation formulas on page 8 of Unit IV may be useful.)

[10] b. Suppose that Z has the standard normal distribution, and let a be a positive constant. Statisticians refer to the distribution of $(Z+a)^2$ as chi-square on one degree of freedom with non-centrality parameter a. Given this fact, determine the distribution of $-2\log\lambda$ if H_1 is true. (Your answer will depend on β and n.)

[20] c. Let $G(\zeta)$ denote the power of the likelihood ratio test as a function of $\zeta = (\zeta_1, \zeta_2)^T$, where ζ_1 is the putative value of α and ζ_2 is the putative value of β. Since $G(\zeta)$ depends on ζ only through ζ_2, we can visualize the power function by graphing $G(\zeta)$ against ζ_2. Do this for $\zeta_2 \in [-2, 2]$ and $n \in \{10, 50, 100\}$. (If using R, you may find that the pchisq function with its ncp option is useful.)

[10] d. What features of your graph are suggestive of unbiasedness and consistency for the likelihood ratio test? (You are not asked to formally prove unbiasedness and consistency.)

[25] 2. Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} f(x;\theta) := \theta x^{\theta-1}$ for $x \in (0,1)$ and $\theta \in \Theta := (0,\infty)$. Let θ_0 be a fixed element of Θ. Propose a uniformly most powerful test of $H_0 : \theta \geq \theta_0$ against $H_1 : \theta < \theta_0$. (Reviewing exercise 9 from Written Assignment 3 may be useful.)

[25] 3. We continue with the scenario from exercise 3 of Written Assignment 4. Suppose that $\theta_0 = 4$ and $n = 5$. Find the actual significance level of the score test whose nominal significance level is 0.05. Describe in careful detail how the score test can be modified so that the actual significance level equals 0.05.