

BST 676 — Spring 2011 — Dr. Charnigo

In-Class Assessment

This assessment is a strictly individual activity. Textbooks, notes, calculators, computers, and technology with Internet access are prohibited. Record that which you want graded in the blue book.

[20] 1. Let $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} f(x; \theta)$ for $\theta \in \Theta \subset \mathbb{R}$.

[10] a. Define “unbiased” in the context of point estimation and in the context of hypothesis testing.

[10] b. Define “consistent” in the context of point estimation and in the context of hypothesis testing.

[30] 2. Let X_1, X_2, \dots, X_n be independent exponential random variables with common mean $\theta \in \Theta := (0, \infty)$.

[10] a. Find the maximum likelihood estimator of θ . (You may assume that you do, in fact, obtain the global maximizer of the likelihood by setting the first derivative of the log likelihood equal to zero.)

[10] b. Without citing the large-sample theory of maximum likelihood estimators, prove that the maximum likelihood estimator of θ is consistent.

[10] c. Exhibit a consistent estimator of $Var_\theta[X_1]$. (This estimator need not be unbiased.)

[30] 3. Continue with the same scenario as in exercise 2.

[10] a. Write out the likelihood ratio statistic λ for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[10] b. Suppose that n is large enough to invoke the large-sample theory of likelihood ratio testing. For what values of λ will you reject H_0 if you desire a test with approximate significance level α ? (You do not need to check the regularity conditions for the large-sample theory of likelihood ratio testing.)

[10] c. Develop a Wald test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ at approximate significance level α .

[20] 4. Continue with the same scenario as in exercises 2 and 3.

[10] a. Develop an approximate $100(1 - \alpha)\%$ Wald interval for θ .

[10] b. Develop an exact $100(1 - \alpha)\%$ pivotal interval for θ . Use the relationship $\chi_{2n}^2 = \text{Gamma}(n, 2)$ to express the endpoints of the interval in terms of \bar{X} and quantiles of the χ_{2n}^2 distribution. (You are not asked to find the shortest such interval. Just exhibit one such interval.)