BST 676 — Spring 2011 — Dr. Charnigo

In-Class Assessment

This assessment is a strictly individual activity. Textbooks, notes, calculators, computers, and technology with Internet access are prohibited. Record that which you want graded in the blue book.

[20] 1. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta)$ for $\theta \in \Theta \subset \mathbb{R}$.

[10] a. Define "unbiased" in the context of point estimation and in the context of hypothesis testing.

[10] b. Define "consistent" in the context of point estimation and in the context of hypothesis testing.

[30] 2. Let $X_1, X_2, \ldots, X_n$ be independent exponential random variables with common mean $\theta \in \Theta := (0, \infty)$.

[10] a. Find the maximum likelihood estimator of θ. (You may assume that you do, in fact, obtain the global maximizer of the likelihood by setting the first derivative of the log likelihood equal to zero.)

[10] b. Without citing the large-sample theory of maximum likelihood estimators, prove that the maximum likelihood estimator of θ is consistent.

[10] c. Exhibit a consistent estimator of $Var_\theta[X_1]$. (This estimator need not be unbiased.)

[30] 3. Continue with the same scenario as in exercise 2.

[10] a. Write out the likelihood ratio statistic λ for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[10] b. Suppose that n is large enough to invoke the large-sample theory of likelihood ratio testing. For what values of λ will you reject H_0 if you desire a test with approximate significance level α? (You do not need to check the regularity conditions for the large-sample theory of likelihood ratio testing.)

[10] c. Develop a Wald test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ at approximate significance level α.

[20] 4. Continue with the same scenario as in exercises 2 and 3.

[10] a. Develop an approximate $100(1-\alpha)\%$ Wald interval for θ.

[10] b. Develop an exact $100(1-\alpha)\%$ pivotal interval for θ. Use the relationship $\chi^2_{2n} = Gamma(n, 2)$ to express the endpoints of the interval in terms of $\bar{X}$ and quantiles of the χ^2_{2n} distribution. (You are not asked to find the shortest such interval. Just exhibit one such interval.)