BST 676 — Spring 2011 — Dr. Charnigo

Final Examination

The Final Examination, a strictly individual activity, is due under my office (CPH 203-B) door at 5 p.m. on Friday 06 May. Office hours are on Monday 02 May from 4 to 6 p.m. and on Friday 06 May from 1 to 3 p.m.

Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} f(x;\theta) := (2\pi\theta)^{-1/2}\exp[-x^2/(2\theta)]$ for $\theta \in \Theta := (0, \infty)$. Let θ_0 be a fixed element of Θ.

[10] 1. What is the uniformly most powerful level α test of $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$? (You may take as given the first five sentences from the Solution to exercise 2c of Written Assignment 5.)

[10] 2. Find the uniformly most accurate interval for θ among all $100(1-\alpha)\%$ intervals of the form $(0, U(\mathbf{X})]$.

[10] 3. Find the uniformly most accurate interval for θ among all $100(1-\alpha)\%$ intervals of the form $[L(\mathbf{X}), \infty)$.

[10] 4. Suppose that we wish to create a two-sided $100(1-\alpha)\%$ interval for θ, starting from the inequality $a < \sum_{i=1}^n X_i^2/\theta < b$. What condition must be placed on a and b?

[10] 5. Suppose, moreover, that we wish to have the shortest possible two-sided $100(1-\alpha)\%$ interval for θ, starting from the inequality $a < \sum_{i=1}^n X_i^2/\theta < b$. What additional condition must be placed on a and b, besides that identified in item 4?

[10] 6. Find the shortest possible two-sided 95% interval for θ, starting from the inequality $a < \sum_{i=1}^n X_i^2/\theta < b$, when $n = 10$.

[10] 7. Develop a Wald test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ at approximate significance level α.

[10] 8. Develop an approximate $100(1-\alpha)\%$ Wald interval for θ.

[10] 9. Show that the approximate $100(1-\alpha)\%$ Wald interval fits expression (21) from Unit VII with $T := \sum_{i=1}^n X_i^2/\theta$.

[10] 10. What is the actual confidence level of the approximate 95% Wald interval when $n = 10$?