BST 676 — Spring 2011 — Dr. Charnigo

Midterm Examination

The Midterm Examination, a strictly individual activity, is due at the end of class on Thursday 10 March. By strictly individual, I mean that you are not to discuss the Midterm Examination with anyone (except me) until after the deadline for its submission.

Suppose that $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta) := x\theta^{-2}\exp[-x\theta^{-1}]1_{\{x\in(0,\infty)\}}$, where $\theta \in \Theta := (0,\infty)$.

[10] 1. Of what commonly encountered family of probability density functions is $f(x;\theta)$ a member? Use known results for members of that family to determine $E_\theta[X_1]$, $Var_\theta[X_1]$, $E_\theta[\sum_{i=1}^n X_i]$, and $Var_\theta[\sum_{i=1}^n X_i]$.

[10] 2. Evaluate the Cramer-Rao lower bound for unbiased estimation of θ.

[20] 3. Show that method of moments and maximum likelihood yield the same estimator of θ and that this estimator — call it $\hat{\theta}$ — is best unbiased.

[10] 4. Describe the large sample behavior of $n^{1/2}(\hat{\theta}^2 - \theta^2)$.

[10] 5. Prove that $\hat{\theta}^2$ is consistent for θ^2.

[15] 6. Show that $\hat{\theta}^2$ is not unbiased for θ^2.

[10] 7. Find an unbiased estimator of θ^2.

[15] 8. Compare $\hat{\theta}^2$ and the unbiased estimator from exercise 7 on mean square error.