BST 676 — Spring 2011 — Dr. Charnigo

Written Assignment 1

Written Assignment 1 is due on Thursday 03 February at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. Suppose that $X_1, X_2, \ldots$ are independent and identically distributed Poisson with mean $\theta \in (0, \infty)$.

[10] a. To what does $\sqrt{n}(\bar{X}_n - \theta)$ converge? (Be sure to justify your answer and to explicitly identify the mode of convergence.)

[10] b. To what does $\bar{X}_n^{-1/2}\sqrt{n}(\bar{X}_n - \theta)$ converge?

[10] c. Use your answer to part b to propose an approximate 95% confidence interval for θ when n is large.

[10] d. Find a real-valued function g such that $\sqrt{n}(g(\bar{X}_n) - g(\theta)) \xrightarrow{L} N(0, 1)$. *Remark.* We refer to g as a *variance stabilizing transformation.*

[10] e. Use your answer to part d to propose an approximate 95% confidence interval for θ when n is large. *Hint.* If $[a, b]$ is a 95% confidence interval for $g(\theta)$, then $[g^{-1}(a), g^{-1}(b)]$ is a 95% confidence interval for θ.

[50] 2. Suppose that $X_1, X_2, \ldots$ are independent and identically distributed with probability density function $f(x; \theta) := \theta x^{\theta-1} 1_{0<x<1}$ and parameter $\theta \in (0, \infty)$.

[10] a. Calculate $E[X_1]$ and $Var[X_1]$.

[10] b. Find a real-valued function h such that $\sqrt{n}(\bar{X}_n - h(\theta))$ converges. (Be sure to justify your answer and to identify that to which the sequence converges.)

[10] c. Find a real-valued function g such that $\sqrt{n}(g(\bar{X}_n) - \theta)$ converges.

[10] d. Use your answer to part c to propose an approximate 95% confidence interval for θ when n is large.

[10] e. When $\bar{X}_n$ is close to 0, your confidence interval will be rather narrow. However, when $\bar{X}_n$ is close to 1, your confidence interval will be rather wide. Can you provide an intuitive explanation for this phenomenon?