BST 676 — Spring 2011 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Thursday 17 February at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta) := \theta(1-\theta)^x$ for $x \in \{0, 1, 2, \ldots\}$ and $\theta \in \Theta := (0, 1]$. Thus, $X_1, X_2, \ldots, X_n$ are geometric with expected value $(1-\theta)/\theta$.

[15] a. Find the maximum likelihood estimate of θ. *Hint*. Consider separately the cases of $\sum_{i=1}^n x_i = 0$ and $\sum_{i=1}^n x_i > 0$.

[10] b. Find the method of moments estimate of θ.

[15] c. Impose the Bayesian prior distribution $p(\theta) := \frac{\Gamma[a+b]}{\Gamma[a]\Gamma[b]}\theta^{a-1}(1-\theta)^{b-1}$ for $\theta \in (0, 1]$, where $a \geq 1$ and $b \geq 1$ are constants. Find the mode of the Bayesian posterior distribution for θ.

[10] d. Continuing from part c, find the mean of the Bayesian posterior distribution for θ.

[50] 2. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(x;\theta) := C(\theta)\exp[x]1_{\{|x|\leq\theta\}}$ for $\theta \in \Theta := (0, \infty)$ and a constant $C(\theta)$ to be determined by you.

[10] a. Determine the constant $C(\theta)$.

[15] b. Find the maximum likelihood estimate of θ.

[10] c. Assuming that $\bar{x} > 0$, show that the method of moments estimate $\hat{\theta}$ satisfies $\hat{\theta}\coth(\hat{\theta}) - 1 = \bar{x}$, where $\coth(\cdot)$ is the hyperbolic cotangent function. What if $\bar{x} \leq 0$?

[15] d. Use R or other software to plot $\hat{\theta}$ as a function of $\bar{x}$ for $\bar{x} \in (0, 5]$. *Hint*. You do not need a nonlinear equation solver. First regard $\bar{x}$ as a function of $\hat{\theta}$, and then reverse the usual roles of the horizontal and vertical axes.