BST 676 — Spring 2011 — Dr. Charnigo

Written Assignment 4

Written Assignment 4 is due on Thursday 31 March at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[20] 1. Suppose that you obtain the following data comparing "high urgency" subjects to "low urgency" subjects in a psychological study assessing whether they are responsive to an experimental stimulus.

	High Urgency	Low Urgency	Row Total
Responsive to Stimulus	9	4	13
Unresponsive to Stimulus	1	6	7
Column Total	10	10	20

Let p_1 denote the fraction of "high urgency" people who would respond to the experimental stimulus in the larger population of which the study sample is representative, and let p_2 denote the fraction of "low urgency" people who would respond to the experimental stimulus in the larger population of which the study sample is representative. Use Fisher's exact test to decide whether to reject $H_0 : p_1 = p_2$ in favor of $H_1 : p_1 \neq p_2$ at significance level 0.05. First obtain an answer using pencil, paper, and hand calculator. Then compare your answer to that acquired from a statistical software package.

[20] 2. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed as exponential random variables with unknown mean parameter $\theta \in (0, \infty)$. Let θ_0 be a known fixed element of $(0, \infty)$.

[10] a. Derive a Wald test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[10] b. Derive a score test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[60] 3. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed with probability density function $f(x; \theta) := 1/\theta$ for $x \in [0, \theta]$, where $\theta \in (0, \infty)$ is an unknown parameter. Let θ_0 be a known fixed element of $(0, \infty)$. Consider testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

[15] a. Use Example #5 from Unit II to evaluate the likelihood ratio test statistic $\lambda := L(\theta_0; \mathbf{X})/L(\hat{\theta}; \mathbf{X})$.

[15] b. Evaluate $P_{\theta_0}[\hat{\theta} \leq a]$ for $a \in [0, \theta_0]$.

[15] c. Evaluate $P_{\theta_0}[(\hat{\theta}/\theta_0)^n \leq a]$ for $a \in [0, 1]$.

[15] d. Find c such that rejection of H_0 with $\lambda < c$ yields a test at significance level 0.05.