BST 676 — Spring 2011 — Dr. Charnigo

Written Assignment 5

Written Assignment 5 is due on Thursday 14 April at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. We continue with the scenario from exercise 3 of Written Assignment 4. Let $G(\zeta)$ denote the power function for the likelihood ratio test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ at significance level 0.05.

[10] a. Evaluate $G(\zeta)$ for $\zeta < \theta_0$.

[15] b. Evaluate $G(\zeta)$ for $\zeta > \theta_0$.

[15] c. Show that the likelihood ratio test is unbiased.

[10] d. Show that the likelihood ratio test is consistent.

[50] 2. Suppose that $X_1, \ldots, X_n \stackrel{iid}{\sim} f(x; \theta) := (2\pi\theta)^{-1/2} \exp[-x^2/(2\theta)]$ for $\theta \in \Theta := (0, \infty)$. Let θ_0 be a fixed element of Θ.

[15] a. Derive the score test of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ with approximate significance level 0.05.

[10] b. Suppose that $\theta_0 = 1$. Plot the actual significance level of the score test as a function of $n \in \{1, 2, \ldots, 100\}$. At what sample size do you feel comfortable using the score test?

[25] c. Find the uniformly most powerful level 0.05 test of $H_0 : \theta \geq \theta_0$ against $H_1 : \theta < \theta_0$. Argue that the uniformly most powerful test is, in essence, an exact one-sided version of the score test.