CPH 931 — Fall 2008 — Dr. Charnigo

Written Assignment 4 Solutions

1a. The hazard ratio (i.e., the hazard of infection for a patient receiving body cleansing divided by the hazard of infection for an otherwise similar patient receiving routine bathing) is $\exp[-\sigma^{-1}\beta_1]$. Body cleansing is a better treatment if this hazard ratio is less than 1, which is true precisely when $\beta_1 > 0$.

Alternatively, we can appeal to the accelerated failure time property of the Weibull model. Time passes $\exp[-\beta_1]$ times as quickly for a patient receiving body cleansing compared to an otherwise similar patient receiving routine bathing. Body cleansing is a better treatment if time passes more slowly for a person receiving it, which is true precisely when $\beta_1 > 0$.

1b. Parameter estimates, standard errors, and p-values are as follows.

Parameter	Estimate	Standard Error	p-value
β_0	4.1177	0.2837	<0.0001
β_1	0.6518	0.3581	0.0688
β_{10}	-0.0239	0.3604	0.9472
σ	1.1888	0.1482	—
σ^{-1}	0.8412	0.1049	—

1c. Display the plot. My code for this is in {WA4931F08SAS.txt}.

1d. For the Weibull model we have

$$S_{x_{1,i},\ldots,x_{k,i}}(t) = \exp[-\exp(\sigma^{-1}\{\log t - \boldsymbol{\beta}\cdot\mathbf{x}_i\})].$$

Setting the left side equal to 0.5, taking logarithms of both sides, multiplying by -1, and taking logarithms again yields

$$\log[-\log 0.5] = \sigma^{-1}\{\log t - \boldsymbol{\beta}\cdot\mathbf{x}_i\}.$$

Multiplying by σ, adding $\boldsymbol{\beta}\cdot\mathbf{x}_i$ to both sides, and exponentiating then gives

$$\exp(\sigma\log[-\log 0.5] + \boldsymbol{\beta}\cdot\mathbf{x}_i) = t.$$

Noting that σ is estimated by 1.1888 and that $\boldsymbol{\beta}\cdot\mathbf{x}_i$ is estimated by $4.1177 + 0.6518z_{1,i} - 0.0239z_{10,i}$, we recover the following estimated median times to infection for the four groups of patients:

$\exp(1.1888\log[-\log 0.5] + 4.1177) = 39.7$ days with routine bathing, no respiratory tract involvement,

$\exp(1.1888\log[-\log 0.5]+4.1177-0.0239) = 38.8$ days with routine bathing, respiratory tract involvement,

$\exp(1.1888\log[-\log 0.5]+4.1177+0.6518) = 76.2$ days with body cleansing, no respiratory tract involvement,

and

$\exp(1.1888\log[-\log 0.5]+4.1177+0.6518-0.0239) = 74.4$ days with body cleansing, respiratory tract involvement.

These estimated median times differ from each other by multiplicative factors of $\exp[\hat{\beta}_1]$ and $\exp[\hat{\beta}_{10}]$, which is a reflection of the accelerated failure time property. For instance, time passing $\exp[-\hat{\beta}_1]$ times as quickly for a patient receiving body cleansing compared to an otherwise similar patient receiving routine bathing is why the estimated median time for the former is $1/\exp[-\hat{\beta}_1] = \exp[\hat{\beta}_1]$ times as large as the estimated median time for the latter.

2a. The hazard ratio (i.e., the hazard of infection for a patient receiving body cleansing divided by the hazard of infection for an otherwise similar patient receiving routine bathing) is $\exp[\beta_1]$. Body cleansing is a better treatment if this hazard ratio is less than 1, which is true precisely when $\beta_1 < 0$.

2b. Parameter estimates, standard errors, and p-values are as follows.

Parameter	Estimate	Standard Error	p-value
β_1	-0.55656	0.29361	0.0580
β_{10}	0.07491	0.30309	0.8048

2c. Display the plot. My code for this is in {WA4931F08SAS.txt}.

3a. The hazard ratio (i.e., the hazard of infection for a patient with excision divided by the hazard of infection for an otherwise similar patient without excision) is $\exp[\beta_{12}]$. Excision is protective against infection if this hazard ratio is less than 1, which is true precisely when $\beta_{12} < 0$.

3b. Parameter estimates, standard errors, and p-values are as follows.

Parameter	Estimate	Standard Error	p-value
β_1	-0.44918	0.29729	0.1308
β_{10}	0.10388	0.30372	0.7323
β_{12}	-0.96324	0.48048	0.0450

3c. The estimate of the hazard ratio is $\exp[-0.96324] = 0.382$. The p-value for testing $H_0 : \beta_{12} = 0$ is 0.0450, but "$\beta_{12} = 0$" is the same as "$\exp[\beta_{12}] = 1$". Hence, the estimate 0.382 is significantly different from 1. Thus, according to part a of this exercise, excision is protective against infection.