CPH 931 — Fall 2009 — Dr. Charnigo

Written Assignment 1 Solutions

1. A number of criticisms are possible. Here are three:

- Not all subjects should have received the medications in the same order. Some subjects should have begun with medication 1, others with medication 2, and the rest with medication 3. (If the washout periods were of insufficient length, then the existing study design would not permit a fair comparison of the medications.)
- Baseline systolic blood pressure scores were not recorded. A baseline systolic blood pressure score should have been recorded for each subject prior to that subject's start on a medication. (Then our analyses could control for baseline variability.)
- Subjects were asked, after two months, to remember how much they exercised in a typical week. Subjects should have been asked to record their exercise activity each week during the two months. (Then our analyses would be less suspectible to recall bias.)

2. For $i = 1, 2, 3$ let μ_i denote the expected systolic blood pressure after two months' treatment with medication i. We perform repeated measures ANOVA to test the null hypothesis that $\mu_1 = \mu_2 = \mu_3$. We obtain $f = 1.37$ (on 2 numerator and 18 denominator degrees of freedom) and $p = 0.2805$, so we may not reject the null hypothesis. Therefore, we may not distinguish among the three medications. [Follow-up tests are not really necessary because we have already accepted the null hypothesis that $\mu_1 = \mu_2 = \mu_3$. However, if follow-up tests are performed, we obtain $p = 0.1840$ for testing $\mu_1 = \mu_2$, $p = 0.9260$ for testing $\mu_1 = \mu_3$, and $p = 0.1573$ for testing $\mu_2 = \mu_3$. These p-values do not include Bonferroni adjustments.]

3. Repeated measures ANOVA does not allow us to control for the subjects' personalities or exercise habits, nor to test for interactions between these factors and treatment. However, linear mixed modeling does allow us to control for the subjects' personalities or exercise habits, as well as to test for interactions between these factors and treatment. [Indeed, exercise 4 showed that there was an interesting story in this data set that could not be revealed by repeated measures ANOVA but that was revealed by linear mixed modeling.] Another point to consider is that if there had been a subject with, for example, a missing value of systolic blood pressure after treatment with medication 3, then repeated measures ANOVA would have disregarded that subject's data entirely. Linear mixed modeling, on the other hand, would have still taken into account the subject's values of systolic blood pressure after treatment with medications 1 and 2.

4a. Put $m := 6$, $X_1 := 1$ when a subject is on medication 1 ($X_1 := 0$ otherwise), $X_2 := 1$ when a subject is on medication 2, $X_3 := 1$ when a subject has Type A personality, $X_4 := 1$ when a subject with Type A personality is on medication 1, $X_5 := 1$ when a subject with Type A personality is on medication 2, and $X_6 :=$ EXER.

4b. We need to test $H_0 : \beta_6 = 0$. The result is $p = 0.0019$, so we reject the null hypothesis and conclude that exercise is somehow related to systolic blood pressure.

4c. [The difference in expected systolic blood pressure of a Type A male on medication 1 versus an otherwise similar Type A male on medication 3 is $\beta_1 + \beta_4$. The difference in expected systolic blood pressure of a Type A male on medication 2 versus an otherwise similar Type A male on medication 3 is $\beta_2 + \beta_5$. The difference in expected systolic blood pressure of a Type A male on medication 1 versus an otherwise similar Type A male on medication 2 is $\beta_1 + \beta_4 - \beta_2 - \beta_5$. These differences are zero if and only if $\beta_1 + \beta_4 = \beta_2 + \beta_5 = 0$.] We need to test $H_0 : \beta_1 + \beta_4 = \beta_2 + \beta_5 = 0$. The result is $p = 0.0012$, so we reject the null hypothesis and conclude that medication is related to systolic blood pressure among Type A males.

4d. [The difference in expected systolic blood pressure of a Type B male on medication 1 versus an otherwise similar Type B male on medication 3 is β_1. The difference in expected systolic blood pressure of a Type B male on medication 2 versus an otherwise similar Type B male on medication 3 is β_2. The difference in expected systolic blood pressure of a Type B male on medication 1 versus an otherwise similar Type B male on medication 2 is $\beta_1 - \beta_2$. These differences are zero if and only if $\beta_1 = \beta_2 = 0$. Recalling that the analogous differences among Type A males are zero if and only if $\beta_1 + \beta_4 = \beta_2 + \beta_5 = 0$, we see that $\beta_1 = \beta_2 = \beta_4 = \beta_5 = 0$ is a requirement for medication to be altogether unrelated to systolic blood pressure.] We need to test $H_0 : \beta_1 = \beta_2 = \beta_4 = \beta_5 = 0$. The result is $p = 0.0017$, so we reject the null hypothesis and conclude that medication is somehow related to systolic blood pressure. [In fact, we know from the previous item that such a relationship exists among Type A males.]

4e. We need to test $H_0 : \beta_4 = \beta_5 = 0$. The result is $p = 0.0013$, so we reject the null hypothesis and conclude that medication interacts with personality.

4f. The expected systolic blood pressure for a Type A male on medication 3 who exercises 4 days a week is $\beta_0 + \beta_3 + 4\beta_6$, while the expected systolic blood pressure for a Type A male on medication 2 who exercises 0 days a week is $\beta_0 + \beta_2 + \beta_3 + \beta_5$. The difference is $4\beta_6 - \beta_2 - \beta_5$, whose estimate is -22.2877. This estimate is significantly different from zero because $H_0 : 4\beta_6 - \beta_2 - \beta_5 = 0$ is rejected with $p < 0.0001$.