CPH 931 — Fall 2009 — Dr. Charnigo

Written Assignment 3 Solutions

1a. Parameter estimates, standard errors, and p-values are as follows.

Parameter	Estimate	Standard Error	p-value
α	1.7042	1.4581	0.2425
β_1	-0.00682	0.7495	0.9927
β_2	0.1931	0.6691	0.7729
β_3	0.2986	0.5589	0.5932
β_4	-0.0923	0.1263	0.4647
α^*	1.9055	2.6896	0.4787
β_1^*	1.5055	1.1798	0.2019
β_2^*	0.2708	1.1657	0.8163
β_3^*	1.9587	1.0825	0.0704
β_4^*	-0.5008	0.2649	0.0587

1b. To answer this question we test $H_0 : \beta_3 = \beta_3^* = 0$ against the complementary alternative. We obtain $\chi^2 = 3.31$ with p-value 0.1908, so $H_0 : \beta_3 = \beta_3^* = 0$ is not rejected and we can reasonably remove HIGHBP from the model. However, for the purpose of this exercise, I retain HIGHBP in what follows.

1c. Such a patient has $x_1 = 1$, $x_2 = x_3 = 0$, and $x_4 = 12$. We calculate

$$\frac{\widehat{p}_{1|\mathbf{x}}}{\widehat{p}_{0|\mathbf{x}}} = \exp[1.7042 - 0.00682 - 0.0923 \times 12] = 1.8036$$

and

$$\frac{\widehat{p}_{2|\mathbf{x}}}{\widehat{p}_{0|\mathbf{x}}} = \exp[1.9055 + 1.5055 - 0.5008 \times 12] = 0.0744,$$

from which we obtain

$$1 + \frac{\widehat{p}_{1|\mathbf{x}}}{\widehat{p}_{0|\mathbf{x}}} + \frac{\widehat{p}_{2|\mathbf{x}}}{\widehat{p}_{0|\mathbf{x}}} = 1 + 1.8036 + 0.0744 = 2.8780 = \frac{1}{\widehat{p}_{0|\mathbf{x}}}.$$

Finally, $\widehat{p}_{0|\mathbf{x}} = 1/2.8780 = 0.3475$, $\widehat{p}_{1|\mathbf{x}} = 0.3475 \times 1.8036 = 0.6268$, and $\widehat{p}_{2|\mathbf{x}} = 0.3475 \times 0.0744 = 0.0259$, so that the estimated probability of being admitted to an ordinary hospital room for such a patient is 0.6268.

1d. Parameter estimates, standard errors, and p-values are as follows.

Parameter	Estimate	Standard Error	p-value
α	-1.2067	1.3153	0.3589
α^*	2.6356	1.3312	0.0477
β_1	0.8053	0.6718	0.2306
β_2	0.3581	0.5886	0.5429
β_3	0.7808	0.5113	0.1268
β_4	-0.1922	0.1146	0.0936

1e. Such a patient has $x_1 = 1$, $x_2 = x_3 = 0$, and $x_4 = 12$. We calculate

$$\widehat{O}_{2|\mathbf{x}} = \exp[-1.2067 + 0.8053 - 0.1922 \times 12] = 0.0667$$

and

$$\widehat{O}_{12|\mathbf{x}} = \exp[2.6356 + 0.8053 - 0.1922 \times 12] = 3.1096,$$

from which we obtain

$$\widehat{p}_{2|\mathbf{x}} = \frac{\widehat{O}_{2|\mathbf{x}}}{1 + \widehat{O}_{2|\mathbf{x}}} = \frac{0.0667}{1 + 0.0667} = 0.0625,$$

$$\widehat{p}_{1|\mathbf{x}} = \left(\widehat{p}_{1|\mathbf{x}} + \widehat{p}_{2|\mathbf{x}}\right) - \widehat{p}_{2|\mathbf{x}} = \left(\frac{\widehat{O}_{12|\mathbf{x}}}{1 + \widehat{O}_{12|\mathbf{x}}}\right) - \widehat{p}_{2|\mathbf{x}} = \frac{3.1096}{1 + 3.1096} - 0.0625 = 0.6942,$$

and

$$\widehat{p}_{0|\mathbf{x}} = 1 - \widehat{p}_{1|\mathbf{x}} - \widehat{p}_{2|\mathbf{x}} = 1 - 0.0625 - 0.6942 = 0.2433,$$

so that the estimated probability of being admitted to an ordinary hospital room for such a patient is 0.6942.

2a. Letting X_1 denote TIME and using otherwise obvious notation, we find that the model-based estimate of the incidence rate ratio comparing day $x_1 + 5$ to day x_1 is

$$\frac{\exp[\widehat{\alpha} + \widehat{\beta}_1(x_1 + 5)]}{\exp[\widehat{\alpha} + \widehat{\beta}_1 x_1]} = \exp[5\widehat{\beta}_1] = \exp[-0.161] = 0.851.$$

This estimate does not depend on x_1, so we obtain 0.851 both in comparing day 15 to day 10 and in comparing day 60 to day 55.

2b. [Display the plot of DAILYINF versus TIME.] The answers to part a do not seem reasonable because the incidence rate ratio appears to change over time. In particular, the incidence rate appears to increase until about day 12, then decrease until about day 23, increase again until about day 29, and then decrease. The incidence rate ratio should be greater than 1 during time intervals when the incidence rate is increasing and less than 1 during time intervals when the incidence rate is decreasing.

2c. Using obvious notation, the model-based estimate of the incidence rate ratio comparing day $x_1 + 5$ to day x_1 is

$$\frac{\exp[\widehat{\alpha} + \widehat{\beta}_1(x_1+5) + \widehat{\beta}_2(x_1+5)^2]}{\exp[\widehat{\alpha} + \widehat{\beta}_1 x_1 + \widehat{\beta}_2 x_1^2]} = \exp[5\widehat{\beta}_1 + 25\widehat{\beta}_2 + 10\widehat{\beta}_2 x_1] = \exp[0.248 - 0.016x_1].$$

With $x_1 = 10$ we obtain $\exp[0.088] = 1.092$ and with $x_1 = 55$ we obtain $\exp[-0.632] = 0.532$. These answers seem more reasonable than those to part a, though even the quadratic model appears to be an oversimplification. (One would need a quartic model to capture an increasing-decreasing-increasing-decreasing pattern in the incidence.)

2d. The quantity

$$\exp[5\beta_1 + 25\beta_2 + 10\beta_2 x_1]$$

is free of x_1 if and only if $\beta_2 = 0$, so we must test $H_0 : \beta_2 = 0$. We have $\chi^2 = 27.75$ with p-value less than 0.0001, so $H_0 : \beta_2 = 0$ is rejected and we declare significantly different the 1.092 and 0.532.

2e. With a multiplicative adjustment of $\sqrt{1.3957}$ to the standard errors in Poisson regression, we still reject $H_0 : \beta_2 = 0$ because $\chi^2 = 19.88$ with p-value less than 0.0001. With negative binomial regression, we reject $H_0 : \beta_2 = 0$ because $\chi^2 = 22.56$ with p-value less than 0.0001.