CPH 931 — Fall 2009 — Dr. Charnigo

Written Assignment 4 Solutions

1a. The hazard for a person with $X = x_i$ is

$$\sigma^{-1} t^{\sigma^{-1}-1} \exp[-\sigma^{-1}(\alpha + \beta x_i)].$$

If $\beta < 0$, then the hazard is larger when x_i is larger. If $\beta > 0$, then the hazard is smaller when x_i is larger. So, if you retained the original convention that $X = 1$ for surgical placement and $X = 2$ for percutaneous placement, then $\beta > 0$ indicates a greater hazard with surgical placement. However, if you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement, then $\beta < 0$ indicates a greater hazard with surgical placement. The former applies if you did not declare placement with a CLASS statement in PROC LIFEREG, while the latter applies if you did.

1b. Parameter estimates, standard errors, and p-values are as follows, assuming you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement. [Show a plot of estimated survival functions for the percutaneous placement and surgical placement groups.]

Parameter	Estimate	Standard Error	p-value
α	4.2197	0.4528	<.0001
β	-0.6227	0.4688	0.1840
σ	1.1374	0.1901	NA

1c. Assuming you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement, the estimated median time to kidney infection for patients with percutaneous catheter placement is obtained by solving the equation

$$0.5 = \exp[-\exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\})]$$

for t. We have

$$\log 0.5 = -\exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\}),$$

$$\log[-\log 0.5] = \widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\},$$

$$\widehat{\sigma}\log[-\log 0.5] = \log t - \widehat{\alpha},$$

$$\widehat{\alpha} + \widehat{\sigma}\log[-\log 0.5] = \log t,$$

and finally

$$\exp(\widehat{\alpha} + \widehat{\sigma}\log[-\log 0.5]) = t.$$

Evaluating the last expression numerically yields 44.8. Likewise, the estimated median time to kidney infection for patients with surgical catheter placement is obtained by solving the equation

$$0.5 = \exp[-\exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha} - \widehat{\beta}\})],$$

which results in

$$\exp(\widehat{\alpha} + \widehat{\beta} + \widehat{\sigma}\log[-\log 0.5]) = t.$$

Evaluating the last expression numerically yields 24.0. The ratio of the estimated median times is

$$\frac{\exp(\widehat{\alpha} + \widehat{\sigma}\log[-\log 0.5])}{\exp(\widehat{\alpha} + \widehat{\beta} + \widehat{\sigma}\log[-\log 0.5])} = \exp[-\widehat{\beta}] = 1.86 = 44.8/24.0.$$

2a. The hazard for a person with $X = x_i$ is

$$\frac{\sigma^{-1}t^{\sigma^{-1}-1}\exp[-\sigma^{-1}(\alpha + \beta x_i)]}{1 + t^{\sigma^{-1}}\exp[-\sigma^{-1}(\alpha + \beta x_i)]},$$

which, upon multiplication of numerator and denominator by $\exp[\sigma^{-1}(\alpha + \beta x_i)]$, can be rewritten as

$$\frac{\sigma^{-1}t^{\sigma^{-1}-1}}{\exp[\sigma^{-1}(\alpha + \beta x_i)] + t^{\sigma^{-1}}}.$$

The rewrite makes clear that the hazard is larger when x_i is larger if $\beta < 0$ and that the hazard is smaller when x_i is larger if $\beta > 0$. So, if you retained the original convention that $X = 1$ for surgical placement and $X = 2$ for percutaneous placement, then $\beta > 0$ indicates a greater hazard with surgical placement. However, if you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement, then $\beta < 0$ indicates a greater hazard with surgical placement.

2b. Parameter estimates, standard errors, and p-values are as follows, assuming you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement. [Show a plot of estimated survival functions for the percutaneous placement and surgical placement groups.] The plot of estimated survival functions based on the log logistic model is generally similar to that based on the Weibull model, although the estimated survival functions based on the log logistic model do not seem to deteriorate as quickly, a perception that will be confirmed by the longer estimated median times in exercise 2c.

Parameter	Estimate	Standard Error	p-value
α	3.8765	0.4507	<.0001
β	-0.4588	0.4984	0.3574
σ	1.0560	0.1762	NA

2c. Assuming you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement, the estimated median time to kidney infection for patients with percutaneous catheter placement is obtained by solving the equation

$$0.5 = \frac{1}{1 + \exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\})}$$

for t. We have

$$\begin{aligned} 2 &= 1 + \exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\}), \\ 1 &= \exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\}), \\ 0 &= \widehat{\sigma}^{-1}\{\log t - \widehat{\alpha}\}, \\ 0 &= \log t - \widehat{\alpha}, \\ \widehat{\alpha} &= \log t, \end{aligned}$$

and finally

$$\exp[\widehat{\alpha}] = t.$$

Evaluating the last expression numerically yields 48.3. Likewise, the estimated median time to kidney infection for patients with surgical catheter placement is obtained by solving the equation

$$0.5 = \frac{1}{1 + \exp(\widehat{\sigma}^{-1}\{\log t - \widehat{\alpha} - \widehat{\beta}\})},$$

which results in

$$\exp[\widehat{\alpha} + \widehat{\beta}] = t.$$

Evaluating the last expression numerically yields 30.5. The ratio of the estimated median times is

$$\frac{\exp[\widehat{\alpha}]}{\exp[\widehat{\alpha} + \widehat{\beta}]} = \exp[-\widehat{\beta}] = 1.58 = 48.3/30.5.$$

The estimated median times based on the log logistic model are slightly longer than those based on the Weibull model.

3a. The hazard for a person with $X = x_i$ is

$$\exp[\alpha(t) + \beta x_i].$$

If $\beta < 0$, then the hazard is smaller when x_i is larger. If $\beta > 0$, then the hazard is larger when x_i is larger. Note that this is the opposite of what we saw in exercises 1a and 2a. So, if you retained the original convention that $X = 1$ for surgical placement and $X = 2$ for percutaneous placement, then $\beta < 0$ indicates a greater hazard with surgical placement. However, if you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement, then $\beta > 0$ indicates a greater hazard with surgical placement.

3b. We had $\widehat{\beta} = 0.61817$ with standard error 0.39813 and p-value 0.1205, assuming you adopted the convention that $X = 1$ for surgical placement and $X = 0$ for percutaneous placement. [Show a plot of estimated survival functions for the percutaneous placement and surgical placement groups.] The plot of estimated survival functions based on the proportional hazards model is noticeably different from those based on the Weibull and log logistic models, first because the estimated survival functions in the present plot are not smooth and second because they deteriorate more rapidly.

3c. We had $\widehat{\beta} = -1.42235$ with standard error 1.03145 and $\widehat{\gamma} = 1.46217$ with standard error 0.58748. The hazard ratio comparing a patient with percutaneous placement to a patient with surgical placement is

$$\frac{\exp[\alpha(t)]}{\exp[\alpha(t) + \beta + \gamma \log t]} = \exp[-\beta - \gamma \log t],$$

which we can estimate as

$$\exp[-\widehat{\beta} - \widehat{\gamma} \log t] = \exp[1.42235 - 1.46217 \log t].$$

When $t = 1$, the estimated hazard ratio is $\exp[1.42235] = 4.15$. When $t = 6$, the estimated hazard ratio is $\exp[-1.19751] = 0.30$. When $t = 12$, the estimated hazard ratio is $\exp[-2.21101] = 0.11$.

3d. The Wald p-value for testing $H_0 : \gamma = 0$ against $H_1 : \gamma \neq 0$ is 0.0128. Along with the results of exercise 3c, this makes us intensely uncomfortable with the familiar proportional hazards model. In fact, this also makes us intensely uncomfortable with the Weibull and log logistic models since neither of these models captures the phenomenon that patients with percutaneous placement seem to be at much higher risk in the first few weeks but not after a few months have passed.