

# Structural Equation Modeling with AMOS

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# Introduction

I provided an overview of structural equation modeling (also called “SEM”) at the Pizza and Pilots session in December 2015. A link to that presentation is included at my website for structural equation modeling.

Without recapitulating that entire presentation, I mention that structural equation modeling may differ from linear regression in one or both of two respects:

First, rather than speaking in terms of “independent” and “dependent” variables, we classify variables as “exogenous” or “endogenous”.

# Introduction

Second, we are often interested in “latent constructs” which are not directly observable. Approximating each one of them, however, will be one or more “observable indicators”.

For today’s workshop, we’ll consider the following (admittedly contrived) scenario:

- There are two latent constructs of interest, cardiovascular precursor (CP) and unhealthy behaviour (UB).
- For CP, there are two observable indicators, blood pressure (BP) and total cholesterol (TC).
- For UB, there are two observable indicators, sedentary days (SD) and dietary intake (DT).

# Introduction

The (fictional) data are contained in the SPSS file at my website for structural equation modeling. To provide a little more detail:

BP is expressed in mmHg, with 140 being borderline high.

TC is expressed in mg/dL, with 200 being borderline high.

SD is the estimated number of days in the last month without at least 15 minutes of physical activity.

DT is the estimated average number, from the last month, of thousands of calories consumed daily.

We also have measurements at each of two time points, so variable names include “1” and “2” suffixes.

# Introduction

We may wonder not only whether CP and UB are related at a given time point but also whether one predicts the other.

For example, is a person with high UB at an earlier time more likely to have high CP at a later time, controlling for that person's level of CP at the earlier time ?

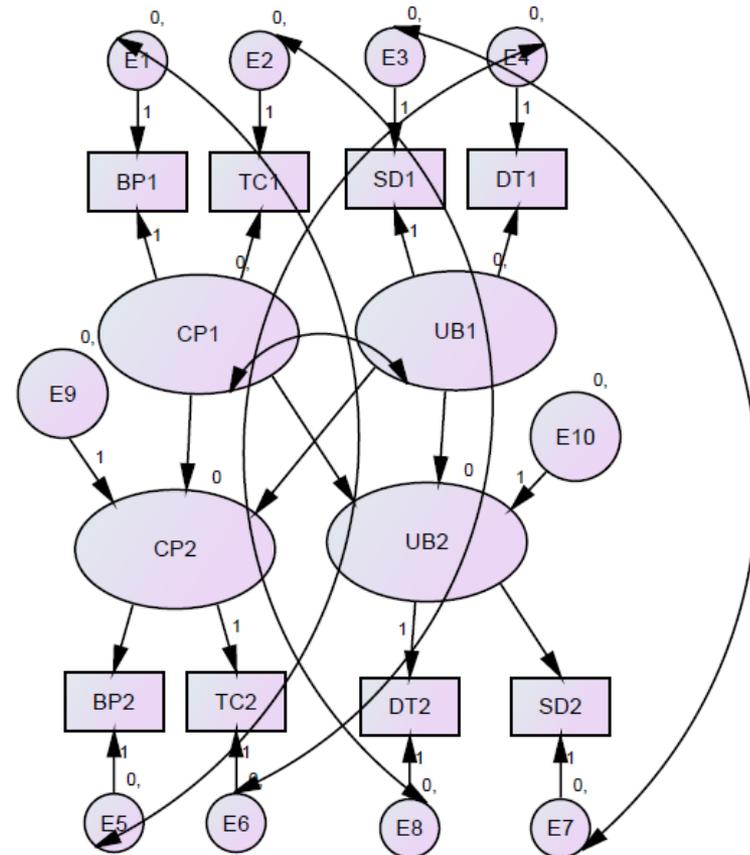
Or, is a person with high CP at an earlier time more likely to have low UB at a later time (perhaps due to some guidance by his/her physician ?), controlling for that person's level of UB at the earlier time ?

# The Model, Some Questions, and Some Answers

At right is a diagram of the structural equation model that I fit.

There are several points for me to address:

1. What do the different shapes (rectangles and ovals) represent ?



# The Model, Some Questions, and Some Answers

2. What do the single arrows represent, and how are they different from the double arrows ?
3. How should  $E_1$  through  $E_{10}$  be interpreted ?
4. What do the o's and 1's represent ?

Note, this diagram is available as the first PDF file at my website for SEM.

# The Model, Some Questions, and Some Answers

The answers are as follows:

1. Rectangles represent observed quantities, while ovals represent unobserved quantities (either latent constructs or “error terms”).
2. A single arrow represents a direct contribution from one quantity to another. So, a given quantity is a linear combination of all the quantities from which it receives arrows. A double arrow represents an assumed nonzero covariance (hence, nonzero correlation).

# The Model, Some Questions, and Some Answers

3. The  $E_1$  through  $E_{10}$  may be called “error terms”. They explain why an observable indicator is not determined solely by the value of the corresponding latent construct; there is an underlying random element as well. They also explain why a latent construct at time 2 cannot be perfectly predicted from the latent constructs at time 1.
4. The  $o$ 's and  $1$ 's represent constraints on regression-type coefficients and variable means, to ensure that the model is identifiable. For example, since adding a  $N(0,1)$  variable is the same as adding twice a  $N(0, \frac{1}{4})$  variable, we have to fix either the variance or the coefficient.

# Results from Modeling

When I fit this model using AMOS, I obtained results as shown in the second PDF file at my website.

The first page of the results file provides a list of observed endogenous variables, unobserved endogenous variables, and unobserved exogenous variables. There are no observed exogenous variables in this model.

The second page of the results file refers to sample moments. There are 8 observed variables, from which we have 8 sample means, 8 sample variances, and 28 sample correlations. These are the 44 sample moments.

## Results from Modeling

There are 33 parameters to estimate in the SEM, leaving  $44 - 33$  or 11 degrees of freedom. There is a test statistic for goodness of fit that follows approximately a chi-square distribution (on 11 degrees of freedom, in this case) if the SEM is correctly specified.

If the test statistic has a value which is compatible with this chi-square distribution, then we accept a null hypothesis that the SEM is correctly specified. Otherwise, we reject that null hypothesis. In our case, the test statistic equals 10.094, for which the corresponding p-value is 0.522. Often, though, the null hypothesis is false and is rejected. Then we may look at the ratio of the test statistic to the degrees of freedom and decide whether the misspecification is severe; a ratio less than 2 is favourable.

## Results from Modeling

The third page of the results file presents estimates of regression-like coefficients relating the latent constructs to each other and to the observable indicators.

The C.R. is the quotient of the estimate by its standard error (sometimes referred to by statisticians as a Wald statistic), and *P* is the accompanying p-value for a test of whether the quantity being estimated is truly nonzero. AMOS uses \*\*\* to represent a p-value too small to display precisely.

# Results from Modeling

However, the regression-like coefficients are difficult to interpret. For example,  $CP_2$  is estimated to equal  $1.462 CP_1 + 1.607 UB_1$  plus an error term. Since we don't observe  $CP_1$  or  $UB_1$ , this equation is opaque.

An alternative perspective is afforded by the standardized regression coefficient estimates, which typically fall between -1 and 1. They are somewhat like partial correlations.

The fourth page of the results file presents intercepts, which are essentially estimated mean levels of the observed indicators. Following that are estimated covariances (and correlations) for each pair of variables connected by a double arrow, then estimated variances for each exogenous variable.

## Results from Modeling

The fifth and sixth pages of results parse total effects into direct and indirect effects, and they also present standardized versions of these quantities.

For example, as noted on the preceding slide, CP<sub>1</sub> makes a direct contribution of 1.462 to CP<sub>2</sub>. Moreover, CP<sub>2</sub> makes a direct contribution of 0.542 to BP<sub>2</sub>. Multiplying 1.462 by 0.542 gives 0.792, which is the indirect contribution of CP<sub>1</sub> to BP<sub>2</sub>.

Note that the quantification of direct and indirect effects would permit one to test hypotheses regarding mediating relationships.

## Results from Modeling

The last three pages of the results file provide numerous indices of model fit. I shall only mention a few of them:

The CMIN/DF, as noted earlier, is the ratio of a test statistic for goodness of fit divided by its degrees of freedom. This ratio assesses the severity of model misspecification. A value less than 2 is favourable, and even a value between 2 and 3 may be tolerable.

The CFI, or comparative fit index, is supplied. A value above 0.95 may be viewed favourably.

## Results from Modeling

The RMSEA, or root mean square error of approximation, is supplied (along with a 90% confidence interval for an underlying quantity). A value less than 0.05 is viewed favourably.

The AIC and BIC don't have easy stand-alone interpretations. However, if you were to fit two different versions of the SEM (for example, by adding/subtracting single arrows or double arrows), you could compare their respective AIC and BIC values. The model with lower AIC and BIC values is favoured.

## Finer Points

Continuing from the preceding slide, in the event that the two models being compared are nested, one may also employ a chi-square test to decide whether the more complex model may be reduced to the simpler model. This is much like a partial F test in linear regression.

For example, if the goodness of fit test statistic equals 20 on 10 degrees of freedom for Model 1, versus 25 on 12 degrees of freedom for Model 2, then the chi-square test statistic for reducing the complex model equals  $25 - 20 = 5$  on  $12 - 10 = 2$  degrees of freedom. Since the level 0.05 critical value for a chi-square distribution on 2 degrees of freedom is 5.99 ( $> 5$ ), we accept the simpler model (on 12 degrees of freedom).

## Finer Points

AMOS will also produce modification indices to suggest various changes which may improve model fit. That didn't happen here, because the SEM I fit was appropriate. However, one often uses the modification indices like one uses forward variable selection in linear regression, except that in SEM one is typically selecting additional single and double arrows rather than variables.

That is, one identifies a single or double arrow whose addition corresponds to the largest modification index, makes that change, re-fits the model, and continues this process until AMOS does not furnish any more suggestions (or, perhaps, until the AIC and BIC suggest discontinuing the process).

## Finer Points

AMOS is generally intuitive, but there are a couple of potential pitfalls for new users.

First, one must remember to Set Analysis Properties and, in particular, Estimate Means and Intercepts.

Second, one must remember to save and name one's workspace before attempting to estimate model parameters; otherwise a cryptic, non-informative error message will appear.

Let me now show how I obtained the preceding results in AMOS.

# Extensions

We will not pursue these today, but I want to mention two extensions which are available in AMOS:

1. If you have missing data, you will need to address this in some manner. One option is to impute the missing values before fitting the structural equation model.
2. You can estimate model parameters differently for each of two groups you wish to compare, such as those persons receiving an intervention versus control participants.