STA 580 — Spring 2011 — Dr. Charnigo

Midterm Examination

Please print your name at the tops of all seven pages. You may use the backs of the pages for scratch work, but all key steps and final answers are to be clearly recorded on the fronts of the pages.

[30] 1. Refer to "Microbiology" and Table 2.18 on page 41.

Let μ_1 and σ_1^2 denote the mean and variance of pod weight in a population of inoculated plants.

Let μ_2 and σ_2^2 denote the mean and variance of pod weight in a population of uninoculated plants.

Let $x_1, \ldots, x_8$ denote the pod weights for the 8 inoculated plants in Table 2.18. Let $y_1, \ldots, y_8$ denote the pod weights for the 8 uninoculated plants in Table 2.18. We have $\sum_{i=1}^{8} x_i = 13.07$, $\sum_{i=1}^{8} x_i^2 = 22.5873$, $\sum_{i=1}^{8} y_i = 8.67$, and $\sum_{i=1}^{8} y_i^2 = 11.2153$.

You may assume that: (i) pod weights are normally distributed in the population of inoculated plants; and, (ii) pod weights are normally distributed in the population of uninoculated plants.

[05] a. Provide point estimates of μ_1 and μ_2.

[05] b. Provide point estimates of σ_1^2 and σ_2^2.

[05] c. Provide a 95% confidence interval for μ_1.

[05] d. Provide a 95% confidence interval for σ_1^2.

[05] e. Using a sample proportion, provide a point estimate of the population proportion of inoculated plants with pod weights less than 1.00.

[05] f. Using normality assumption (i) and the point estimates of μ_1 and σ_1^2, provide a second point estimate of the population proportion of inoculated plants with pod weights less than 1.00. Which point estimate — this one or the one from item e — do you find more credible?

[30] 2. Continue with the same scenario as in exercise 1. Assume that the two samples are independent.

[05] a. Test $H_0 : \sigma_1^2 = 1.00$ against $H_1 : \sigma_1^2 < 1.00$ at level $\alpha = 0.05$.

[05] b. Test $H_0 : \mu_1 = 1.50$ against $H_1 : \mu_1 \neq 1.50$ at level $\alpha = 0.05$.

[05] c. Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at level $\alpha = 0.05$.

[05] d. Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ at level $\alpha = 0.05$.

[05] e. Estimate the sample size (inoculated plants) required for 90% power to reject $H_0 : \mu_1 = 1.50$ in favor of $H_1 : \mu_1 \neq 1.50$ at level $\alpha = 0.05$.

[05] f. Assuming a sample size of 30 (inoculated plants), estimate the power to reject $H_0 : \mu_1 = 1.50$ in favor of $H_1 : \mu_1 \neq 1.50$ at level $\alpha = 0.05$.

[20] 3. Individuals exhibiting a certain set of symptoms are screened for H1N1. Suppose that: (i) the screening test results in a positive diagnosis for 95% of individuals who really do have H1N1; (ii) the screening test results in a positive diagnosis for 10% of individuals who really do not have H1N1; and, (iii) 30% of individuals exhibiting the set of symptoms have H1N1.

Let D denote the event that a randomly selected individual exhibiting the set of symptoms is diagnosed positively, and let H denote the event that this individual has H1N1.

[15] a. Supply each of the following probabilities.

- [03] Find $P(D|H)$.

- [03] Find $P(D|\bar{H})$.

- [03] Find $P(\bar{H})$.

- [03] Find $P(D)$.

- [03] Find $P(\bar{H}|D)$.

[05] b. Suppose that 2 unrelated individuals exhibiting the set of symptoms are screened for H1N1. What is the probability that there will be at least 1 positive diagnosis?

[20] 4. Identify the best answer for each item and record its letter to the left of the item. Multiple letters, illegible letters, and non-responses will be considered incorrect answers. Scoring for this exercise will be as follows: 7 correct answers, 20 points; 6 correct answers, 18 points; 5 correct answers, 15 points; 4 correct answers, 12 points; 3 correct answers, 9 points; 2 correct answers, 6 points; 1 correct answer, 3 points; 0 correct answers, 0 points.

i. Suppose that a 95% confidence interval for a population mean μ is 10 to 20. Which of the following statements is most obviously not true?

a. We reject $H_0 : \mu = 11$ in favor of $H_1 : \mu \neq 11$ at level $\alpha = 0.05$.

b. We do not reject $H_0 : \mu = 15$ in favor of $H_1 : \mu \neq 15$ at level $\alpha = 0.05$.

c. We reject $H_0 : \mu = 8$ in favor of $H_1 : \mu \neq 8$ at level $\alpha = 0.05$.

d. The 90% confidence interval for μ is 12 to 18.

e. The 99% confidence interval for μ is 7 to 23.

ii. Which of the following statements about a p-value is not true?

a. If we are conducting a hypothesis test at level $\alpha = 0.05$, a p-value of less than 0.05 warrants rejection of the null hypothesis.

b. If we are conducting a hypothesis test at level $\alpha = 0.05$, a p-value of less than 0.01 warrants rejection of the null hypothesis.

c. A p-value is the probability of obtaining a result at least as extreme in its opposition to the null hypothesis as the result actually observed, this probability being calculated under the supposition that the null hypothesis is true.

d. A p-value is the conditional probability that the null hypothesis is true given the result actually observed.

iii. Suppose that X is a binomial random variable based on $n = 10$ trials with success probability $p = 0.40$. Which of the following statements is not true?

a. The probability that $4 \leq X \leq 5$ can be obtained by adding the probability mass function evaluated at 4 to the probability mass function evaluated at 5.

b. The probability that $4 \leq X \leq 5$ can be obtained by subtracting the cumulative distribution function evaluated at 3 from the cumulative distribution function evaluated at 5.

c. The probability that $4 \leq X \leq 5$ can be obtained by finding the area under the curve of the probability density function from 4 to 5.

d. The probability that $4 \leq X \leq 5$ is larger than the probability that $X = 4$.

e. The probability that $4 \leq X \leq 5$ is smaller than the probability that $4 \leq X \leq 6$.

iv. Which of the following responses would have been appropriate if I had asked you in exercise 1 to construct a 95% confidence interval for the population proportion of inoculated plants with pod weights less than 1.00?

a. Use the formula $\hat{p}_1 \pm 1.96\sqrt{\hat{p}_1(1-\hat{p}_1)/8}$.

b. Use the formula $\hat{p}_1 \pm 2.365\sqrt{\hat{p}_1(1-\hat{p}_1)/8}$.

c. The interval should be 0 to 0 since none of the inoculated plants in the sample had a pod weight less than 1.00.

d. None of responses a, b, or c would have been appropriate.

v. Let Z have the standard normal distribution. Which of the following statements about Z is true?

a. The probability that $Z = 1.645$ is 0.95.

b. The probability that $Z = 1.96$ is 0.95.

c. The probability that $Z > 1.645$ is 0.05.

d. The probability that $Z > 1.96$ is 0.05.

e. The probability that $1.645 \leq Z \leq 1.96$ is 0.05.

vi. Which of the following statements about assumption (i) in exercise 1 is true?

a. Assumption (i) is necessary to do part a of exercise 1.

b. Assumption (i) is necessary to do part b of exercise 1.

c. Assumption (i) is necessary to do part d of exercise 1.

d. Assumption (i) is necessary to do both part b and part d of exercise 1.

vii. Which of the following statements does not follow from the Central Limit Theorem? (All symbols below have their usual meanings.)

a. For large n, $\bar{X}$ is approximately normal.

b. For large n, $(n-1)S^2/\sigma^2$ is approximately chi-square on $(n-1)$ degrees of freedom.

c. For large n, $\hat{P}$ is approximately normal.

d. For large n, we are justified in constructing a confidence interval of the form $\bar{x} \pm z_{1-\alpha/2}s/\sqrt{n}$ for a population mean μ.

e. For large n, we are justified in testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ by rejecting H_0 if $|\bar{x} - \mu_0|/(s/\sqrt{n}) > z_{1-\alpha/2}$.