STA 570 — Spring 2012 — Dr. Charnigo

Written Assignment 2 Solutions

1a. From Written Assignment 1 we know that $\bar{x} = 112.6$, $s = 34.2$, and $n = 49$. The $100(1-\alpha)\%$ "small sample" confidence interval for μ is

$$\bar{x} \pm t_{n-1,1-\alpha/2} s/\sqrt{n}.$$

Putting $\alpha = 0.10$ and noting that $t_{n-1,1-\alpha/2} = t_{48,0.95} = 1.677$, we obtain

$$112.6 \pm 8.2, \text{ which is } 104.4 \text{ to } 120.8.$$

With a "large" sample size, $t_{n-1,1-\alpha/2}$ could be replaced by $z_{1-\alpha/2}$, and we would not need to assume normality of the low density lipoprotein measurements.

1b. We conduct a level α "small sample" test of $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ by constructing the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and comparing its absolute value to $t_{n-1,1-\alpha/2}$. In this case, with $\mu_0 = 130$ and $\alpha = 0.05$, we have

$$t = \frac{112.6 - 130}{34.2/\sqrt{49}} = \frac{-17.4}{4.89} = -3.56,$$

whose absolute value is greater than $t_{48,0.975} = 2.011$. Therefore we reject $H_0 : \mu = 130$ in favor of $H_1 : \mu \neq 130$.

1c. The general formula for (approximate) power in testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ at level α is

$$\Phi\left(-z_{1-\alpha/2} + \frac{|\mu_0 - \mu_1|\sqrt{n}}{\sigma}\right).$$

We have $\alpha = 0.05$, so that $-z_{1-\alpha/2} = -z_{0.975} = -1.96$. We also have $\mu_0 = 130$ and $n = 75$. Taking $\mu_1 = 112.6$ and $\sigma = 34.2$, as we have no compelling reason to do otherwise, we find that the power is

$$\Phi\left(-1.96 + \frac{|130 - 112.6|\sqrt{75}}{34.2}\right) = \Phi(2.446) = 99.3\%.$$

1d. The general formula for (approximate) sample size in testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ at level α, when one desires power $1-\beta$, is

$$n = \frac{\sigma^2 (z_{1-\beta} + z_{1-\alpha/2})^2}{(\mu_0 - \mu_1)^2}.$$

With $1 - \beta = 0.95$ we have $z_{1-\beta} = z_{0.95} = 1.645$. So the required sample size is

$$\frac{34.2^2 (1.645 + 1.96)^2}{(130 - 112.6)^2} \approx 51.$$

1e. The $100(1-\alpha)\%$ confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} \text{ to } \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}.$$

Putting $\alpha = 0.05$ and noting that $\chi^2_{n-1,1-\alpha/2} = \chi^2_{48,0.975} = 69.02$, $\chi^2_{n-1,\alpha/2} = \chi^2_{48,0.025} = 30.75$, we obtain

$$\frac{(48)34.2^2}{69.02} \text{ to } \frac{(48)34.2^2}{30.75}, \text{ which is 813.4 to 1826.}$$

With a "large" sample size, the same formula would be used, and we would still need to assume normality of the low density lipoprotein measurements.

2a. We can easily calculate that $\bar{x} = 3.57$, $s = 3.37$, and $n = 200$. The $100(1-\alpha)\%$ "large sample" confidence interval for μ is

$$\bar{x} \pm z_{1-\alpha/2} s/\sqrt{n}.$$

Putting $\alpha = 0.05$ and noting that $z_{1-\alpha/2} = z_{0.975} = 1.96$, we obtain

$$3.57 \pm 0.47, \text{ which is 3.10 to 4.04.}$$

2b. We conduct a level α "large sample" test of $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ by constructing the test statistic

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and comparing it to $z_{1-\alpha}$. In this case, with $\mu_0 = 3$ and $\alpha = 0.01$, we have

$$z = \frac{3.57 - 3}{3.37/\sqrt{200}} = \frac{0.57}{0.238} = 2.39,$$

which is greater than $z_{0.99} = 2.33$. Therefore we reject $H_0 : \mu = 3$ in favor of $H_1 : \mu > 3$.

2c. The general formula for power in testing $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ at level α is

$$\Phi\left(-z_{1-\alpha} + \frac{|\mu_0 - \mu_1|\sqrt{n}}{\sigma}\right).$$

[Note that μ_1 must be larger than μ_0 to apply this formula, for otherwise $H_1 : \mu > \mu_0$ would not be true.] We have $\alpha = 0.01$, so that $-z_{1-\alpha} = -z_{0.99} = -2.33$. We also have $\mu_0 = 3$ and $n = 300$. Taking $\mu_1 = 3.57$ and $\sigma = 3.37$, as we have no compelling reason to do otherwise, we find that the power is

$$\Phi\left(-2.33 + \frac{|3 - 3.57|\sqrt{300}}{3.37}\right) = \Phi(0.60) \approx 72.6\%.$$

The power would be greater at significance level $\alpha = 0.05$ since the criterion for rejection of a null hypothesis is less stringent at significance level $\alpha = 0.05$ than at significance level $\alpha = 0.01$.

2d. The general formula for sample size in testing $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ at level α, when one desires power $1 - \beta$, is

$$n = \frac{\sigma^2(z_{1-\beta} + z_{1-\alpha})^2}{(\mu_0 - \mu_1)^2}.$$

[Note that μ_1 must be larger than μ_0 to apply this formula, for otherwise $H_1 : \mu > \mu_0$ would not be true.] With $1 - \beta = 0.80$ we have $z_{1-\beta} = z_{0.80} = 0.842$. So the required sample size is

$$\frac{3.37^2(0.842 + 2.33)^2}{(3 - 3.57)^2} \approx 352.$$

The sample size for 80% power would be lesser at significance level $\alpha = 0.05$ since 352 subjects would yield greater than 80% power at significance level $\alpha = 0.05$.

2e. We can easily calculate that $\hat{p} = 127/200 = 0.635$ and $n = 200$. The $100(1 - \alpha)\%$ "large sample" confidence interval for p is

$$\hat{p} \pm z_{1-\alpha/2}\sqrt{\hat{p}(1 - \hat{p})/n}.$$

Putting $\alpha = 0.05$ and noting that $z_{1-\alpha/2} = z_{0.975} = 1.96$, we obtain

$$0.635 \pm 0.067, \text{ which is } 0.568 \text{ to } 0.702.$$

Since $n\hat{p}(1-\hat{p}) = 46.36$ is well in excess of 10, we are comfortable regarding the sample size as large enough to use the above formula for the confidence interval.