STA 570 — Spring 2012 — Dr. Charnigo

Written Assignment 5 Solutions

1a. We have $\bar{y}_1 = 2.600$, $s_1^2 = 2.489$, $\bar{y}_2 = 3.600$, $s_2^2 = 4.711$, $\bar{y}_3 = 5.800$, $s_3^2 = 9.067$, and $\bar{\bar{y}} = 4.000$. Hence, Between SS is

$$10 \times (2.600^2 + 3.600^2 + 5.800^2) - 30 \times (4.000)^2 = 53.6.$$

Also, Within SS is

$$(10 - 1) \times (2.489 + 4.711 + 9.067) = 146.4.$$

[Quoting SAS output on this and subsequent items is permissible. Note that s_3^2 is more than three times as large as s_1^2, so that the assumption of equal population variances is dubious. Despite this, as wellas the possibility of non-normally distributed lengths of stay within each population, we proceed with all of the subsequent items.]

1b. Between MS is $53.6/(3-1) = 26.8$ and Within MS is $146.4/(30-3) = 5.42$, so $f = 26.8/5.42 = 4.94 > 3.35 = f_{2,27,0.95}$. [Since $f_{2,27,0.95}$ does not appear in Table 9 of your textbook, an approximation is satisfactory for the purpose of this exercise. Alternatively, one may note that the SAS-reported p-value of 0.0148 is less than 0.05.] Thus, we reject H_0 in favor of the complementary alternative.

1c. To test $H_0 : \mu_3 = \mu_1$ we calculate

$$t = \frac{5.800 - 2.600}{\sqrt{5.42(1/10 + 1/10)}} = 3.07.$$

Since $t_{27,0.975} = 2.05$, we reject $H_0 : \mu_3 = \mu_1$. [The SAS-reported p-value is 0.0048.]

To test $H_0 : \mu_3 = \mu_2$ we calculate

$$t = \frac{5.800 - 3.600}{\sqrt{5.42(1/10 + 1/10)}} = 2.11.$$

Since $t_{27,0.975} = 2.05$, we reject $H_0 : \mu_3 = \mu_2$. [The SAS-reported p-value is 0.0440.]

1d. The t statistics from 1c are unaltered, but now the original critical value of $t_{27,0.975} = 2.05$ must be replaced by $t_{27,0.9875} = 2.37$. [Since $t_{27,0.9875}$ does not appear in Table 5 of your textbook, an approximation is satisfactory for the purpose of this exercise. Notice, however, that approximation via the nearest column in Table 5 is typically not as reliable as approximation via the nearest row.] We still reject $H_0 : \mu_3 = \mu_1$, but we no longer reject $H_0 : \mu_3 = \mu_2$. [The SAS-reported p-values from part c can be doubled and then compared to 0.05.]

1e. Since $\mu_3 - 0.5\mu_1 - 0.5\mu_2 = 0$ is the same as $\mu_3 = (\mu_1 + \mu_2)/2$, this null hypothesis says that mean length of stay among patients transferred from more than 50 miles away is the same as the average of mean length of stay among patients transferred from less than 25 miles away and mean length of stay among patients transferred from between 25 and 50 miles away. [The latter is an approximation to mean length of stay among patients transferred from less than 50 miles away.]

1f. We calculate

$$t = \frac{5.800 - 0.5 \times 2.600 - 0.5 \times 3.600}{\sqrt{5.42(1/10 + 0.25/10 + 0.25/10)}} = \frac{2.700}{0.902} = 2.99.$$

With a Scheffe adjustment, the critical value is not $t_{27,0.975} = 2.05$ but rather $\sqrt{(3-1)f_{2,27,0.95}} = 2.59$. Even so, we are able to reject H_0. [Since $f_{2,27,0.95}$ does not appear in Table 9 of your textbook, an approximate critical value is satisfactory for the purpose of this exercise.]

1g. We have $n_1 = n_2 = n_3 = 10$, $N = 30$, $r_1 = 107.0$, $r_2 = 145.5$, and $r_3 = 212.5$. Since there are numerous ties (2 six-way, 3 four-way, and 1 two-way), the test statistic is

$$\frac{\frac{12}{30(31)} \times (107.0^2/10 + 145.5^2/10 + 212.5^2/10) - 3(31)}{1 - \frac{2(6^3-6)+3(4^3-4)+1(2^3-2)}{30^3-30}} = 7.52.$$

Since the critical value is $\chi^2_{2,0.95} = 5.99$, we reject H_0. [The SAS-reported p-value is 0.0232.]

2a. We have $a = 3$, $b = 2$, balanced data with $n = 5$, $\bar{y}_{11} = 2.600$, $\bar{y}_{12} = 2.600$, $\bar{y}_{21} = 3.400$, $\bar{y}_{22} = 3.800$, $\bar{y}_{31} = 5.400$, $\bar{y}_{32} = 6.200$, $\bar{y}_{1.} = 2.600$, $\bar{y}_{2.} = 3.600$, $\bar{y}_{3.} = 5.800$, $\bar{y}_{.1} = 3.800$, $\bar{y}_{.2} = 4.200$, and $\bar{y}_{..} = 4.000$. Also, $s^2_{11} = 3.300$, $s^2_{12} = 2.300$, $s^2_{21} = 4.300$, $s^2_{22} = 6.200$, $s^2_{31} = 9.300$, and $s^2_{32} = 10.700$. Thus, we have

$$SST = 4(3.300+2.300+4.300+6.200+9.300+10.700)+5(2.600^2+2.600^2+3.400^2+3.800^2+5.400^2+6.200^2)$$
$$-30(4.000)^2 = 200.0,$$
$$SSA = 10(2.600^2 + 3.600^2 + 5.800^2) - 30(4.000)^2 = 53.6,$$
$$SSB = 15(3.800^2 + 4.200^2) - 30(4.000)^2 = 1.2,$$
$$SSAB = 5(2.600^2 + 2.600^2 + 3.400^2 + 3.800^2 + 5.400^2 + 6.200^2) - 30(4.000)^2 - 53.6 - 1.2 = 0.8,$$

and

$$SSE = 200.0 - 53.6 - 1.2 - 0.8 = 144.4.$$

2b. In testing for nonzero interaction effects we have

$$f_{AB} = \frac{0.8/2}{144.4/24} = 0.07.$$

Since this does not exceed $f_{2,24,0.95} = 3.40$, we do not reject the null hypothesis of zero interaction effects. [The SAS-reported p-value is 0.9359.]

In testing for nonzero main effects from distance we have

$$f_A = \frac{53.6/2}{144.4/24} = 4.45.$$

Since this exceeds $f_{2,24,0.95} = 3.40$, we reject the null hypothesis of zero main effects from distance. [The SAS-reported p-value is 0.0226.]

In testing for nonzero main effects from clot-busting therapy we have

$$f_B = \frac{1.2/1}{144.4/24} = 0.20.$$

Since this does not exceed $f_{1,24,0.95} = 4.26$, we do not reject the null hypothesis of zero main effects from clot-busting therapy. [The SAS-reported p-value is 0.6592.]