STA 580 — Fall 2007 — Dr. Charnigo

Midterm Examination

Please print your name at the tops of all six pages. You may use the backs of the pages for scratch work, but all key steps and final answers are to be clearly recorded on the fronts of the pages.

[25] 1. Refer to "Ophthalmology" and Table 8.17 on page 342, but suppose that results have now been acquired for two more patients: one in the cataractous eyes group and one in the normal eyes group. Patient #7 in the cataractous eyes group had a median gray level of 154, while patient #7 in the normal eyes group had a median gray level of 164.

Let μ_1 and σ_1^2 denote the mean and variance in median gray level for cataractous eyes. Let p_1 denote the proportion of cataractous eyes with median gray level above 160. Let μ_2, σ_2^2, and p_2 be defined analogously for normal eyes.

In what follows you may proceed as if median gray levels for cataractous eyes were normally distributed and median gray levels for healthy eyes were normally distributed. You may take for granted that $\sum_{i=1}^{7} x_i = 1017$, $\sum_{i=1}^{7} x_i^2 = 150411$, $\sum_{i=1}^{7} y_i = 1178$, and $\sum_{i=1}^{7} y_i^2 = 199452$, where the x_i are sample values for patients in the cataractous eyes group and the y_i are sample values for patients in the normal eyes group.

[05] a. Provide point estimates of μ_1 and σ_1^2.

[05] b. Provide point estimates of μ_2 and σ_2^2.

[05] c. Provide a 95% confidence interval for μ_1.

[05] d. Provide a 95% confidence interval for σ_1^2.

[05] e. Provide point estimates of p_1 and p_2.

[25] 2. Continue with the same scenario as in exercise 1.

[05] a. Test $H_0 : \mu_1 = 150$ against $H_1 : \mu_1 \neq 150$ at level $\alpha = 0.05$.

[05] b. Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at level $\alpha = 0.05$.

[05] c. Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ at level $\alpha = 0.05$.

[05] d. Estimate the sample size (patients in the cataractous eyes group) required for 80% power to reject $H_0 : \mu_1 = 150$ in favor of $H_1 : \mu_1 \neq 150$ at level $\alpha = 0.05$.

[05] e. Assuming a sample size of 25 (patients in the cataractous eyes group), estimate the power to reject $H_0 : \mu_1 = 150$ in favor of $H_1 : \mu_1 \neq 150$ at level $\alpha = 0.05$.

[25] 3. Individuals exhibiting a certain set of symptoms are screened for a viral infection. Suppose that: (i) the screening test results in a positive diagnosis for 92% of individuals who really do have the infection; (ii) the screening test results in a positive diagnosis for 16% of individuals who really do not have the infection; and, (iii) 8.0% of individuals exhibiting the set of symptoms have the infection.

Let D denote the event that a randomly selected individual exhibiting the set of symptoms is diagnosed positively, and let I denote the event that this individual has the infection.

[15] a. Supply each of the following probabilities.

- Find $P(D|I)$.

- Find $P(D|\bar{I})$.

- Find $P(I)$.

- Find $P(D)$.

- Find $P(I|D)$.

[10] b. Suppose that 150 *infected* individuals exhibiting the set of symptoms are to be screened. What is the approximate probability that the screening test will yield fewer than 15 incorrect diagnoses?

[25] 4. Mark each of the following statements as true or false.

T F [05] a. The normality assumption in the statement of exercise 1 would have been necessary to complete part b of exercise 2 even if there had been 225 patients in the cataractous eyes group and 225 patients in the normal eyes group.

T F [05] b. Generally speaking, a p-value less than the significance level of a hypothesis test implies rejection of the null hypothesis.

T F [05] c. Refer to Table 8.22 on page 345. Viewing the Week 0 measurements as one set of sample values and the Week 1 measurements as another set of sample values, the samples may be regarded as independent.

T F [05] d. If part e of exercise 1 had instead asked you to test $H_0 : p_1 = 0.50$ against $H_1 : p_1 > 0.50$ at level $\alpha = 0.05$, you would have calculated a z statistic and rejected the null hypothesis if the z statistic had exceeded 1.645.

T F [05] e. If part a of exercise 2 had instead asked you to test $H_0 : \mu_1 = 150$ against $H_1 : \mu_1 > 150$ at level $\alpha = 0.05$, you would have calculated a t statistic and rejected the null hypothesis if the t statistic had exceeded 1.895.