STA 580 — Fall 2007 — Dr. Charnigo

Practice Midterm Examination

Please print your name at the tops of all six pages. You may use the backs of the pages for scratch work, but all key steps and final answers are to be clearly recorded on the fronts of the pages. *For the actual midterm examination you must bring your textbook or, at least, photocopies of the exercise sets from Chapters 6, 7, and 8 plus Tables 3, 5, 6, and 9.*

[25] 1. Refer to "Ophthalmology" and Table 7.10 on page 294, but suppose (contrary to the textbook's description) that the data were obtained from two independent samples.

Let μ_1 and σ_1^2 denote the mean and variance of intraocular pressure for glaucoma patients on medications A and B. Also, let p_1 denote the fraction of glaucoma patients on medications A and B who have intraocular pressure greater than 21. Let μ_2, σ_2^2, and p_2 be defined analogously for glaucoma patients on medication C.

In what follows you may proceed as if intraocular pressure were normally distributed within each of the two populations determined by these treatment strategies. Also, you may take for granted that $\sum_{i=1}^{10} x_i = 177.5$, $\sum_{i=1}^{10} x_i^2 = 3210.25$, $\sum_{i=1}^{10} y_i = 178.0$, and $\sum_{i=1}^{10} y_i^2 = 3304.50$, where the x_i are sample values for patients on medications A and B and the y_i are sample values for patients on medication C.

[05] a. Provide point estimates of μ_1 and σ_1^2.

[05] b. Provide point estimates of μ_2 and σ_2^2.

[05] c. Provide a 95% confidence interval for μ_1.

[05] d. Provide a 95% confidence interval for σ_1^2.

[05] e. Noting that $\hat{p}_1 = 0.1$, a student tries to calculate a 95% confidence interval for p_1 as $0.1 \pm 1.96\sqrt{0.1(1-0.1)/10}$, which simplifies to $[-0.09, 0.29]$. The student is nonplussed: p_1 cannot be a negative number, yet he cannot find any arithmetic mistakes in his computations. What is the student's error?

[25] 2. Continue with the same scenario as in exercise 1. Note that $f_{9,9,0.025} = 0.248$, $f_{9,9,0.05} = 0.315$, $f_{9,9,0.95} = 3.179$, $f_{9,9,0.975} = 4.026$, $f_{9,18,0.025} = 0.270$, $f_{9,18,0.05} = 0.338$, $f_{9,18,0.95} = 2.456$, and $f_{9,18,0.975} = 2.929$.

[05] a. Test $H_0 : \mu_1 = 18$ against $H_1 : \mu_1 \neq 18$ at level $\alpha = 0.05$.

[05] b. Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at level $\alpha = 0.05$.

[05] c. Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ at level $\alpha = 0.05$.

[05] d. Estimate the sample size (glaucoma patients on medications A and B) required for 90% power to reject $H_0 : \mu_1 = 18$ in favor of $H_1 : \mu_1 \neq 18$ at level $\alpha = 0.05$.

[05] e. Assuming a sample size of 36 (glaucoma patients on medications A and B), estimate the power to reject $H_0 : \mu_1 = 18$ in favor of $H_1 : \mu_1 \neq 18$ at level $\alpha = 0.05$.

[25] 3. Individuals exhibiting a certain set of symptoms are screened for a viral infection. Suppose that: (i) the screening test results in a positive diagnosis for 75% of individuals who really do have the infection; (ii) the screening test results in a negative diagnosis for 75% of individuals who really do not have the infection; and, (iii) 10% of individuals exhibiting the set of symptoms have the infection.

Let D denote the event that a randomly selected individual exhibiting the set of symptoms is diagnosed positively, and let I denote the event that this individual has the infection.

[15] a. Supply each of the following probabilities.

- Find $P(D|I)$.

- Find $P(D|\bar{I})$.

- Find $P(I)$.

- Find $P(D)$.

- Find $P(I|D)$.

[10] b. Suppose that 200 individuals exhibiting the set of symptoms are to be screened. What is the approximate probability that at least 40 positive diagnoses are made?

[25] 4. Mark each of the following statements as true or false.

T F [05] a. The normality assumption in the statement of exercise 1 would not have been necessary to complete part c of that exercise if there had been 300 glaucoma patients in each sample.

T F [05] b. Generally speaking, the p-value is the probability that the null hypothesis is true.

T F [05] c. Refer to Table 8.24 on page 345. Viewing the High-ozone change scores as one set of sample values and the Low-ozone change scores as another set of sample values, the samples may be regarded as independent.

T F [05] d. The statement $\Phi(1.96) = 0.9750$ means that $P(Z = 1.96) = 0.9750$, where Z is a standard normal random variable.

T F [05] e. The normality assumption in the statement of exercise 1 would not have been necessary to complete part d of that exercise if there had been 300 glaucoma patients in each sample.