STA 580 — Fall 2008 — Dr. Charnigo

Written Assignment 5 Solutions

1a. We have $\bar{y}_1 = 4.84$, $s_1^2 = 1.033$, $\bar{y}_2 = 7.12$, $s_2^2 = 1.217$, $\bar{y}_3 = 4.44$, $s_3^2 = 1.343$, $\bar{y}_4 = 4.70$, $s_4^2 = 0.965$, $\bar{y}_5 = 7.38$, $s_5^2 = 1.167$, and $\bar{\bar{y}} = 5.696$. Hence, Between SS is

$$5 \times (4.84^2 + 7.12^2 + 4.44^2 + 4.70^2 + 7.38^2) - 25 \times (5.696)^2 = 40.8296.$$

[Direct quote from SAS output is permissible.] Also, Within SS is

$$(5-1) \times (1.033 + 1.217 + 1.343 + 0.965 + 1.167) = 22.900.$$

[Again, direct quote from SAS output is permissible.]

1b. Between MS is $40.8296/(5-1) = 10.2074$ and Within MS is $22.900/(25-5) = 1.1450$, so $f = 10.2074/1.1450 = 8.91 > 2.87 = f_{4,20,0.95}$. Thus, we reject H_0 in favor of the complementary alternative at level 0.05. [If using SAS, noting that the p-value 0.0003 is less than 0.05 is sufficient to reject H_0.]

1c. To test $H_0 : \mu_5 = \mu_3$ we calculate

$$t = \frac{7.38 - 4.44}{\sqrt{1.1450(1/5 + 1/5)}} = 4.34.$$

Since $t_{20,0.975} = 2.09$, we are able to reject $H_0 : \mu_5 = \mu_3$ at level 0.05. [If using SAS, noting that the p-value 0.0003 is less than 0.05 is sufficient to reject H_0.]

To test $H_0 : \mu_5 = \mu_4$ we calculate

$$t = \frac{7.38 - 4.70}{\sqrt{1.1450(1/5 + 1/5)}} = 3.96.$$

Since $t_{20,0.975} = 2.09$, we are able to reject $H_0 : \mu_5 = \mu_4$ at level 0.05. [If using SAS, noting that the p-value 0.0008 is less than 0.05 is sufficient to reject H_0.]

To test $H_0 : \mu_4 = \mu_3$ we calculate

$$t = \frac{4.70 - 4.44}{\sqrt{1.1450(1/5 + 1/5)}} = 0.38.$$

Since $t_{20,0.975} = 2.09$, we are unable to reject $H_0 : \mu_4 = \mu_3$ at level 0.05. [If using SAS, noting that the p-value 0.7049 is greater than 0.05 is sufficient to withhold rejection of H_0.]

1d. The t statistics from 1c are unaltered, but now the original critical value of $t_{20,0.975} = 2.09$ must be replaced by $t_{20,0.9917} = 2.61$. [In grading we will accept $t_{20,0.99} = 2.53$ as an approximation to $t_{20,0.9917}$, although the approximation is obviously not too good!] Even so, our decisions to reject $H_0 : \mu_5 = \mu_3$, reject $H_0 : \mu_5 = \mu_4$, and accept $H_0 : \mu_4 = \mu_3$ are unaltered. [The p-values are $3 \times 0.0003 = 0.0009$, $3 \times 0.0008 = 0.0023$ (with less rounding the 0.0008 is actually 0.00077), and 1.0000 respectively.]

1e. The mean duration of relief for an aspirin-based pain reliever is $(1/2)\mu_1 + (1/2)\mu_2$, while the mean duration of relief for a non-aspirin-based pain reliever is $(1/3)\mu_3 + (1/3)\mu_4 + (1/3)\mu_5$. Hence, we wish to test $H_0 : (1/2)\mu_1 + (1/2)\mu_2 - (1/3)\mu_3 - (1/3)\mu_4 - (1/3)\mu_5 = 0$ against the complementary alternative

$H_1 : (1/2)\mu_1 + (1/2)\mu_2 - (1/3)\mu_3 - (1/3)\mu_4 - (1/3)\mu_5 \neq 0$. [Since $(1/2)\mu_1 + (1/2)\mu_2 - (1/3)\mu_3 - (1/3)\mu_4 - (1/3)\mu_5 = 0$ is logically equivalent to $3\mu_1 + 3\mu_2 - 2\mu_3 - 2\mu_4 - 2\mu_5 = 0$, in grading we will accept answers of $H_0 : 3\mu_1 + 3\mu_2 - 2\mu_3 - 2\mu_4 - 2\mu_5 = 0$ and $H_1 : 3\mu_1 + 3\mu_2 - 2\mu_3 - 2\mu_4 - 2\mu_5 \neq 0$. In this case the numerator and denominator of the t statistic in 1f are multiplied by six, but the factors of six cancel each other out.]

1f. We calculate

$$t = \frac{(1/2)4.84 + (1/2)7.12 - (1/3)4.44 - (1/3)4.70 - (1/3)7.38}{\sqrt{1.1450\{(1/2)^2/5 + (1/2)^2/5 + (-1/3)^2/5 + (-1/3)^2/5 + (-1/3)^2/5\}}} = \frac{0.473}{0.437} = 1.08.$$

[Direct quote from SAS output is permissible.] With a Scheffe adjustment, the critical value is not $t_{20,0.975} = 2.09$ but rather $\sqrt{(5-1)f_{4,20,0.95}} = 3.39$. Hence, we are not able to reject H_0 after a Scheffe adjustment is made. [Actually, we would not have been able to reject H_0 without a Scheffe adjustment. But the p-value without such an adjustment would have been 0.2915, whereas the p-value with it is 0.8787.]

1g. We have $n_1 = n_2 = n_3 = n_4 = n_5 = 5$, $N = 25$, $r_1 = 44.5$, $r_2 = 97.5$, $r_3 = 37.5$, $r_4 = 42.5$, and $r_5 = 103$. Since there are three two-way ties, the test statistic is

$$\frac{\frac{12}{25(26)} \times (44.5^2/5 + 97.5^2/5 + 37.5^2/5 + 42.5^2/5 + 103^2/5) - 3(26)}{1 - \frac{3(2^3-2)}{25^3-25}} = 15.46.$$

[Direct quote from SAS output is permissible and, in fact, preferred.] Since the critical value is $\chi^2_{4,0.95} = 9.49$, we reject H_0 at level 0.05. [If using SAS, noting that the p-value 0.0038 is less than 0.05 is sufficient to reject H_0.]

2a and 2b. We have $a = b = 2$, balanced data with $n = 5$, $\bar{y}_{11} = 10.34$, $\bar{y}_{12} = 8.50$, $\bar{y}_{21} = 8.30$, $\bar{y}_{22} = 5.98$, $\bar{y}_{1.} = 9.42$, $\bar{y}_{2.} = 7.14$, $\bar{y}_{.1} = 9.32$, $\bar{y}_{.2} = 7.24$, and $\bar{y}_{..} = 8.28$. Also, $s^2_{11} = 3.168$, $s^2_{12} = 3.215$, $s^2_{21} = 2.595$, and $s^2_{22} = 2.067$. Thus, we have

$$SST = 4(3.168 + 3.215 + 2.595 + 2.067) + 5(10.34^2 + 8.50^2 + 8.30^2 + 5.98^2) - 20(8.28)^2 = 92.092,$$

$$SSA = 10(9.42^2 + 7.14^2) - 20(8.28)^2 = 25.992,$$

$$SSB = 10(9.32^2 + 7.24^2) - 20(8.28)^2 = 21.632,$$

$$SSAB = 5(10.34^2 + 8.50^2 + 8.30^2 + 5.98^2) - 20(8.28)^2 - 25.992 - 21.632 = 0.288,$$

and

$$SSE = 92.092 - 25.992 - 21.632 - 0.288 = 44.180.$$

[Direct quote from SAS output is permissible and, in fact, preferred.]

2c. In testing for nonzero interaction effects we have

$$f_{AB} = \frac{0.288/1}{44.180/16} = 0.10.$$

Since this does not exceed $f_{1,16,0.95} = 4.49$, we do not reject the null hypothesis of no nonzero interaction effects. [If using SAS, noting that the p-value 0.7509 is greater than 0.05 is sufficient to withhold rejection of this null hypothesis.]

In testing for nonzero main effects from counseling we have

$$f_A = \frac{25.992/1}{44.180/16} = 9.41.$$

Since this exceeds $f_{1,16,0.95} = 4.49$, we reject the null hypothesis of no nonzero main effects from counseling. [If using SAS, noting that the p-value 0.0074 is less than 0.05 is sufficient to reject this null hypothesis.]

In testing for nonzero main effects from medication we have

$$f_B = \frac{21.632/1}{44.180/16} = 7.83.$$

Since this exceeds $f_{1,16,0.95} = 4.49$, we reject the null hypothesis of no nonzero main effects from medication. [If using SAS, noting that the p-value 0.0129 is less than 0.05 is sufficient to reject this null hypothesis.]

[These results can be interpreted as follows. Overall, children who received counseling (resp., medication) did better than children who did not receive counseling (resp., medication). Moreover, the benefit of counseling (resp., medication) for children who also received medication (resp., counseling) was similar to the benefit of counseling (resp., medication) for children who did not receive medication (resp., counseling).]