STA 580 — Spring 2009 — Dr. Charnigo

Written Assignment 4 Solutions

1a. We have $\hat{p}_1 = 70/200 = 0.3500$, $\hat{p}_2 = 85/200 = 0.4250$, and $\hat{p} = 155/400 = 0.3875$. The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/200 + 1/200)}} = \frac{-0.0750}{0.0487} = -1.54,$$

which does not exceed 1.96 in absolute value. So, we accept H_0 at level 0.05. [The p-value is 0.124.]

1b. We have $E_{11} = 155 \times 200/400 = 77.5$, $E_{21} = 77.5$, $E_{12} = 122.5$, and $E_{22} = 122.5$. The test statistic is

$$\chi^2 = \frac{(70-77.5)^2}{77.5} + \frac{(130-122.5)^2}{122.5} + \frac{(85-77.5)^2}{77.5} + \frac{(115-122.5)^2}{122.5} = 2.37,$$

which does not exceed $3.84 = \chi^2_{1,0.95}$. So, we accept H_0 at level 0.05. [The p-value is 0.124.]

2a. [Exhibit the box plots.] The distribution of white blood count measurements does not appear positively skewed either among medical patients or among surgical patients. The box plot for medical patients is somewhat remarkable because the sample median and sample 75^{th} percentile coincide; however, this is undoubtedly an artifact of the small sample size. Interestingly, the distribution of white blood count measurements would appear positively skewed if we were to combine medical patients and surgical patients.

2b. The sum of ranks is 89 for the medical patients and 236 for the surgical patients. If the median white blood count for medical patients were equal to that of surgical patients, then we would have expected a rank sum of $9(9+16+1)/2 = 117$ for the medical patients and a rank sum of $16(9+16+1)/2 = 208$ for the surgical patients. The absolute value of the difference between 89 and 117, or between 236 and 208, is 28. Using Equation 9.7, and noting the presence of several ties (two four-way, two three-way, four two-way), we obtain

$$z = \frac{28 - 1/2}{\sqrt{\left(\frac{9(16)}{12}\right)\left[26 - \frac{60+60+24+24+6+6+6+6}{25(24)}\right]}} = 1.57.$$

Since the test statistic does not exceed 1.96 in absolute value, we do not conclude that median white blood count differs between the two service groups. [The p-value is 0.117.]

2c. After logarithmic transformation the sample means are 1.801 for the medical patients and 2.075 for the surgical patients. The sample variances are 0.0625 and 0.2211, yielding a pooled variance estimate of

$$\frac{0.0625 \times 8 + 0.2211 \times 15}{23} = 0.1659.$$

So, the test statistic is

$$t = \frac{1.801 - 2.075}{\sqrt{0.1659(1/9 + 1/16)}} = -1.61.$$

Since the test statistic does not exceed $2.07 = t_{23,0.975}$ in absolute value, we do not conclude that mean log-transformed white blood count differs between the two service groups. [The p-value is 0.120.]

2d. Each sample should have size 30 since

$$\frac{(0.0625 + 0.2211)(1.960 + 0.842)^2}{(1.801 - 2.075)^2} = 29.66$$

rounds up to 30. [Since the hypothesis test itself would employ a t reference distribution rather than a z reference distribution, in practice we might be conservative and bump the 30 up to a 31 or a 32.]

3a. There are $n = 23$ nonzero difference scores. Using Equation 9.1 we determine that more than $16.7 = 23/2 + 1/2 + 1.96\sqrt{23/4}$ or fewer than $6.3 = 23/2 - 1/2 - 1.96\sqrt{23/4}$ positive difference scores would permit the conclusion of a nonzero median difference between weight change in the control period and weight change in the lack of consistency period. Since 16 of the nonzero difference scores are positive, we are not entitled to that conclusion. [The p-value is 0.093.]

3b. The sum of the ranks for the positive difference scores is 211.5. If the median difference between weight change in the control period and weight change in the lack of consistency period were zero, then we would have expected a rank sum of $138 = 23(24)/4$ for the positive difference scores. Using Equation 9.5, and noting the presence of one two-way tie, we obtain

$$z = \frac{|211.5 - 138| - 1/2}{\sqrt{23(24)(47)/24 - 6/48}} = 2.22.$$

Since 2.22 exceeds 1.96, we may conclude that the median difference between weight change in the control period and weight change in the lack of consistency period is nonzero. [The p-value is 0.026.]

3c. The test statistic is

$$t = \frac{4.83}{9.33/\sqrt{23}} = 2.48.$$

Since 2.48 exceeds $2.07 = t_{22,0.975}$, we may conclude that the mean difference between weight change in the control period and weight change in the lack of consistency period is nonzero. [The p-value is 0.021.]

3d. The sign test, which uses the least amount of information from the data, does not distinguish between the control and lack of consistency periods. The paired t-test, which uses the greatest amount of information from the data, does distinguish between the control and lack of consistency periods. The signed rank test, which uses an intermediate amount of information from the data, also distinguishes between the control and lack of consistency periods.

The difference scores in Table 9.13 appear close enough to normally distributed that we are comfortable using the paired t-test. When we are comfortable using the paired t-test, it is the best choice since it uses the greatest amount of information from the data and hence gives us our best chance at rejecting a false null hypothesis.