STA 580 — Spring 2009 — Dr. Charnigo

Written Assignment 5 Solutions

1a. We have $\bar{y}_1 = 8.6$, $s_1^2 = 6.2^2 = 38.44$, $\bar{y}_2 = 5.3$, $s_2^2 = 5.4^2 = 29.16$, $\bar{y}_3 = 4.9$, $s_3^2 = 7.0^2 = 49.00$, $\bar{y}_4 = 1.1$, $s_4^2 = 6.5^2 = 42.25$, and $\bar{\bar{y}} = 4.975$. Hence, Between SS is

$$20 \times (8.6^2 + 5.3^2 + 4.9^2 + 1.1^2) - 80 \times (4.975)^2 = 565.35.$$

Also, Within SS is

$$(20 - 1) \times (38.44 + 29.16 + 49.00 + 42.25) = 3018.15.$$

1b. Between MS is $565.35/(4-1) = 188.45$ and Within MS is $3018.15/(80-4) = 39.7125$, so $f = 188.45/39.7125 = 4.745 > 2.725 = f_{3,76,0.95}$. [Since $f_{3,76,0.95}$ does not appear in Table 9 of your textbook, an approximation anywhere between $2.68 = f_{3,120,0.95}$ and $2.76 = f_{3,60,0.95}$ will be considered satisfactory for the purpose of this exercise.] Thus, we reject H_0 in favor of the complementary alternative.

1c. To test $H_0 : \mu_1 = \mu_2$ we calculate

$$t = \frac{8.6 - 5.3}{\sqrt{39.7125(1/20 + 1/20)}} = 1.656.$$

Since $t_{76,0.975} = 1.992$, we are unable to reject $H_0 : \mu_1 = \mu_2$. [Since $t_{76,0.975}$ does not appear in Table 5 of your textbook, an approximation anywhere between $t_{120,0.975} = 1.980$ and $t_{60,0.975} = 2.000$ will be considered satisfactory for the purpose of this exercise.]

To test $H_0 : \mu_1 = \mu_3$ we calculate

$$t = \frac{8.6 - 4.9}{\sqrt{39.7125(1/20 + 1/20)}} = 1.857.$$

Since $t_{76,0.975} = 1.992$, we are unable to reject $H_0 : \mu_1 = \mu_3$.

1d. The t statistics from 1c are unaltered, but now the original critical value of $t_{76,0.975} = 1.992$ must be replaced by $t_{76,0.9875} = 2.287$. [Since $t_{76,0.9875}$ does not appear in Table 5 of your textbook, an approximation anywhere between $(1/2)t_{120,0.975} + (1/2)t_{120,0.99} = 2.169$ and $t_{60,0.99} = 2.390$ will be considered satisfactory for the purpose of this exercise.] Our decisions not to reject $H_0 : \mu_1 = \mu_2$ and not to reject $H_0 : \mu_1 = \mu_3$ are unaltered. [Can you explain why a Bonferroni adjustment will *never* change a "do not reject" decision to a "reject" decision?]

1e. The expected benefit of counseling for meditation, if one receives counseling for weight reduction, is $\mu_1 - \mu_2$. The expected benefit of counseling for meditation, if one does not receive counseling for weight reduction, is $\mu_3 - \mu_4$. If $\mu_1 - \mu_2 = \mu_3 - \mu_4$, then $\mu_1 - \mu_2 - \mu_3 + \mu_4 = 0$. Hence, we wish to test $H_0 : \mu_1 - \mu_2 - \mu_3 + \mu_4 = 0$ against the complementary alternative $H_1 : \mu_1 - \mu_2 - \mu_3 + \mu_4 \neq 0$. [Multiplying $\mu_1 - \mu_2 - \mu_3 + \mu_4$ through by any constant, such as 1/2 or 1/4, is acceptable. In this case, the numerator and denominator of the t statistic in exercise 1f are multiplied by that same constant.]

1f. We calculate

$$t = \frac{8.6 - 5.3 - 4.9 + 1.1}{\sqrt{39.7125\{1/20 + 1/20 + 1/20 + 1/20\}}} = \frac{-0.5}{2.818} = -0.177.$$

With a Scheffe adjustment, the critical value is not $t_{76,0.975} = 1.992$ but rather $\sqrt{(4-1)f_{3,76,0.95}} = 2.859$. Hence, we are unable to reject H_0. [Since $f_{3,76,0.95}$ does not appear in Table 9 of your textbook, an approximate critical value anywhere between $\sqrt{(4-1)f_{3,120,0.95}} = 2.835$ and $\sqrt{(4-1)f_{3,60,0.95}} = 2.877$ will be considered satisfactory for the purpose of this exercise.]

1g. We have $n_1 = n_2 = n_3 = n_4 = 20$, $N = 80$, $r_1 = 1051.5$, $r_2 = 821.5$, $r_3 = 694$, and $r_4 = 673$. Since there are nine two-way ties and one three-way tie, the test statistic is

$$\frac{\frac{12}{80(81)} \times (1051.5^2/20 + 821.5^2/20 + 694^2/20 + 673^2/20) - 3(81)}{1 - \frac{9(2^3-2)+(3^3-3)}{80^3-80}} = 8.398.$$

Since the critical value is $\chi^2_{3,0.95} = 7.81$, we reject H_0.

2a. We have $a = b = 2$, balanced data with $n = 20$, $\bar{y}_{11} = 8.6$, $\bar{y}_{12} = 5.3$, $\bar{y}_{21} = 4.9$, $\bar{y}_{22} = 1.1$, $\bar{y}_{1.} = 6.95$, $\bar{y}_{2.} = 3.00$, $\bar{y}_{.1} = 6.75$, $\bar{y}_{.2} = 3.20$, and $\bar{y}_{..} = 4.975$. Also, $s^2_{11} = 38.44$, $s^2_{12} = 29.16$, $s^2_{21} = 49.00$, and $s^2_{22} = 42.25$. Thus, we have

$$SST = 19(38.44 + 29.16 + 49.00 + 42.25) + 20(8.6^2 + 5.3^2 + 4.9^2 + 1.1^2) - 80(4.975)^2 = 3583.5,$$

$$SSA = 40(6.95^2 + 3.00^2) - 80(4.975)^2 = 312.05,$$

$$SSB = 40(6.75^2 + 3.20^2) - 80(4.975)^2 = 252.05,$$

$$SSAB = 20(8.6^2 + 5.3^2 + 4.9^2 + 1.1^2) - 80(4.975)^2 - 312.05 - 252.05 = 1.25,$$

and

$$SSE = 3583.5 - 312.05 - 252.05 - 1.25 = 3018.15.$$

2b. In testing for nonzero interaction effects we have

$$f_{AB} = \frac{1.25/1}{3018.15/76} = 0.031.$$

Since this does not exceed $f_{1,76,0.95} = 3.967$, we do not reject the null hypothesis of zero interaction effects. [Since $f_{1,76,0.95}$ does not appear in Table 9 of your textbook, an approximation anywhere between $3.92 = f_{1,120,0.95}$ and $4.00 = f_{1,60,0.95}$ will be considered satisfactory for the purpose of this exercise.]

In testing for nonzero main effects from counseling for weight reduction we have

$$f_A = \frac{312.05/1}{3018.15/76} = 7.858.$$

Since this exceeds $f_{1,76,0.95} = 3.967$, we reject the null hypothesis of zero main effects from counseling for weight reduction.

In testing for nonzero main effects from counseling for meditation we have

$$f_B = \frac{252.05/1}{3018.15/76} = 6.347.$$

Since this exceeds $f_{1,76,0.95} = 3.967$, we reject the null hypothesis of zero main effects from counseling for meditation.

2c. The null hypothesis proposed in part e of exercise 1 states that there is zero interaction between counseling for meditation and counseling for weight reduction, since nonzero interaction between two factors entails that the effect of one factor (e.g., counseling for meditation) depend on the level of the other factor (e.g., counseling for weight reduction).