STA 580 — Fall 2008 — Dr. Charnigo

Midterm Examination

Please print your name at the tops of all six pages. You may use the backs of the pages for scratch work, but all key steps and final answers are to be clearly recorded on the fronts of the pages.

[25] 1. Refer to "Nutrition" and Table 8.16 on page 341.

Let μ_1 and σ_1^2 denote the mean and variance in the number of colds over a 12-month period in a population of prisoners receiving Vitamin C. Let p_1 denote the proportion of such prisoners with 3 or fewer colds.

Let μ_2 and σ_2^2 denote the mean and variance in the number of colds over a 12-month period in a population of prisoners receiving placebo. Let p_2 denote the proportion of such prisoners with 3 or fewer colds.

Letting $x_1, \ldots, x_{10}$ denote the numbers of colds over a 12-month period for the 10 prisoners receiving Vitamin C in Table 8.16 and letting $y_1, \ldots, y_{10}$ denote the numbers of colds over a 12-month period for the 10 prisoners receiving placebo in Table 8.16, we find that $\sum_{i=1}^{10} x_i = 33$, $\sum_{i=1}^{10} x_i^2 = 131$, $\sum_{i=1}^{10} y_i = 57$, and $\sum_{i=1}^{10} y_i^2 = 341$.

In what follows you may proceed as if the number of colds were normally distributed in both populations.

[05] a. Provide point estimates of μ_1 and σ_1^2.

[05] b. Provide point estimates of μ_2 and σ_2^2.

[05] c. Provide a 95% confidence interval for μ_1.

[05] d. Provide a 95% confidence interval for σ_1^2.

[05] e. Provide point estimates of p_1 and p_2.

[25] 2. Continue with the same scenario as in exercise 1. Some of the following information may be useful: $f_{9,9,.025} = 0.248$, $f_{9,9,.05} = 0.315$, $f_{9,9,.95} = 3.179$, $f_{9,9,.975} = 4.026$, $f_{10,10,.025} = 0.269$, $f_{10,10,.05} = 0.336$, $f_{10,10,.95} = 2.978$, $f_{10,10,.975} = 3.717$, $f_{9,18,.025} = 0.270$, $f_{9,18,.05} = 0.338$, $f_{9,18,.95} = 2.456$, $f_{9,18,.975} = 2.929$.

[05] a. Test $H_0 : \mu_1 = 4$ against $H_1 : \mu_1 < 4$ at level $\alpha = 0.05$.

[05] b. Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at level $\alpha = 0.05$.

[05] c. Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ at level $\alpha = 0.05$.

[05] d. Estimate the sample size (prisoners in the Vitamin C group) required for 90% power to reject $H_0 : \mu_1 = 4$ in favor of $H_1 : \mu_1 < 4$ at level $\alpha = 0.05$.

[05] e. Assuming a sample size of 20 (prisoners in the Vitamin C group), estimate the power to reject $H_0 : \mu_1 = 4$ in favor of $H_1 : \mu_1 < 4$ at level $\alpha = 0.05$.

[25] 3. Individuals exhibiting a certain set of symptoms are screened for a myocardial infarction. Suppose that: (i) the screening test results in a positive diagnosis for 90% of individuals who really do have a myocardial infarction; (ii) the screening test results in a negative diagnosis for 95% of individuals who really do not have a myocardial infarction; and, (iii) 20% of individuals exhibiting the set of symptoms have a myocardial infarction.

Let D denote the event that a randomly selected individual exhibiting the set of symptoms is diagnosed positively, and let I denote the event that this individual has a myocardial infarction.

[15] a. Supply each of the following probabilities.

- Find $P(D|I)$.

- Find $P(D|\bar{I})$.

- Find $P(I)$.

- Find $P(D)$.

- Find $P(I|D)$.

[10] b. Suppose that 300 individuals exhibiting the set of symptoms are screened for a myocardial infarction. What is the approximate probability that 70 or fewer of these individuals actually have a myocardial infarction?

[25] 4. Mark each of the following statements as true or false.

T F [05] a. Even though you completed exercises 1 and 2 as if the number of colds were normally distributed in both populations, strictly speaking the number of colds cannot be normally distributed in either population since the number of colds is a discrete rather than a continuous random variable.

T F [05] b. Referring to part c of exercise 1: the probability that μ_1 falls between 2.0 and 4.5 is something more than 95% but something less than 99%.

T F [05] c. Power is the probability that you incorrectly accept the null hypothesis when it is false.

T F [05] d. Referring to part a of exercise 2: even though you would need a computer to calculate the p-value exactly, your decision to accept the null hypothesis at level 0.05 implies that the p-value cannot be less than 0.05.

T F [05] e. Referring to part e of exercise 1: if asked to test $H_0 : p_1 = 0.2$ against $H_1 : p_1 > 0.2$ at level $\alpha = 0.05$, you would calculate the test statistic $t = (0.5 - 0.2)/\sqrt{0.2(1 - 0.2)/10}$ and then reject H_0 if t were greater than the critical value 1.833.