STA 580 — Spring 2009 — Dr. Charnigo

Midterm Examination

Please print your name at the tops of all six pages. You may use the backs of the pages for scratch work, but all key steps and final answers are to be clearly recorded on the fronts of the pages.

[30] 1. Refer to "Obstetrics" and Table 8.18 on page 342.

Let μ_1 and σ_1^2 denote the mean and variance of birthweight in a population of babies born to mothers receiving drug A. Let p_1 denote the proportion of such babies with birthweights less than 6.00 lbs.

Let μ_2 and σ_2^2 denote the mean and variance of birthweight in a population of babies born to mothers receiving placebo. Let p_2 denote the proportion of such babies with birthweights less than 6.00 lbs.

Letting $x_1, \ldots, x_{15}$ denote the birthweights for the 15 babies in Table 8.18 born to mothers receiving drug A and letting $y_1, \ldots, y_{15}$ denote the birthweights for the 15 babies in Table 8.18 born to mothers receiving placebo, we find that $\sum_{i=1}^{15} x_i = 106.20$, $\sum_{i=1}^{15} x_i^2 = 763.22$, $\sum_{i=1}^{15} y_i = 93.90$, and $\sum_{i=1}^{15} y_i^2 = 600.73$.

You may assume that: (i) birthweights are normally distributed in the population of babies born to mothers receiving drug A; and, (ii) birthweights are normally distributed in the population of babies born to mothers receiving placebo.

[05] a. Provide point estimates of μ_1 and μ_2.

[05] b. Provide point estimates of σ_1^2 and σ_2^2.

[05] c. Provide point estimates of σ_1 and σ_2.

[05] d. Provide a 95% confidence interval for μ_1.

[05] e. Provide a 95% confidence interval for σ_1^2.

[05] f. Provide point estimates of p_1 and p_2.

[30] 2. Continue with the same scenario as in exercise 1. Assume that the two samples are independent. Some of the following information may be useful: $f_{14,14,.025} = 0.336, f_{14,14,.05} = 0.403, f_{14,14,.95} = 2.484, f_{14,14,.975} = 2.979, f_{15,15,.025} = 0.349, f_{15,15,.05} = 0.416, f_{15,15,.95} = 2.403, f_{15,15,.975} = 2.862, f_{14,28,.025} = 0.364, f_{14,28,.05} = 0.431, f_{14,28,.95} = 2.064, f_{14,28,.975} = 2.374, f_{15,30,.025} = 0.378, f_{15,30,.05} = 0.445, f_{15,30,.95} = 2.015, f_{15,30,.975} = 2.307.$

[05] a. Test $H_0 : \sigma_1^2 = 4.00$ against $H_1 : \sigma_1^2 < 4.00$ at level $\alpha = 0.05$.

[05] b. Test $H_0 : \mu_1 = 6.75$ against $H_1 : \mu_1 > 6.75$ at level $\alpha = 0.05$.

[05] c. Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at level $\alpha = 0.05$.

[05] d. Test $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ at level $\alpha = 0.05$.

[05] e. Estimate the sample size (babies born to mothers receiving drug A) required for 80% power to reject $H_0 : \mu_1 = 6.75$ in favor of $H_1 : \mu_1 > 6.75$ at level $\alpha = 0.05$.

[05] f. Assuming a sample size of 30 (babies born to mothers receiving drug A), estimate the power to reject $H_0 : \mu_1 = 6.75$ in favor of $H_1 : \mu_1 > 6.75$ at level $\alpha = 0.05$.

[25] 3. Individuals exhibiting a certain set of symptoms are screened for a myocardial infarction. Suppose that: (i) the screening test results in a positive diagnosis for 80% of individuals who really do have a myocardial infarction; (ii) the screening test results in a positive diagnosis for 10% of individuals who really do not have a myocardial infarction; and, (iii) 15% of individuals exhibiting the set of symptoms have a myocardial infarction.

Let D denote the event that a randomly selected individual exhibiting the set of symptoms is diagnosed positively, and let I denote the event that this individual has a myocardial infarction.

[15] a. Supply each of the following probabilities.

- Find $P(D|I)$.

- Find $P(D|\bar{I})$.

- Find $P(I)$.

- Find $P(D)$.

- Find $P(I|D)$.

[10] b. Suppose that 250 individuals exhibiting the set of symptoms are screened for a myocardial infarction. What is the approximate probability that there are 50 or more positive diagnoses?

[15] 4. Mark each of the following statements as true or false.

T F [03] a. Suppose that we are conducting a hypothesis test at level $\alpha = 0.05$. If the p-value is less than 0.05, then we reject the null hypothesis.

T F [03] b. Referring to part c of exercise 1: constructing a confidence interval for μ_1 required assumption (i) on page 1 of this examination but did not require assumption (ii), and even assumption (i) would not have been necessary if the sample size (babies born to mothers receiving drug A) had been sufficiently large.

T F [03] c. Referring to part d of exercise 1: the probability is 0.95 that σ_1^2 falls inside the confidence interval that you constructed.

T F [03] d. Referring to part e of exercise 1: if I had instead asked you to construct a 95% confidence interval for p_1, the most appropriate response would have been for you to evaluate $\hat{p}_1 \pm 1.96\sqrt{\hat{p}_1(1 - \hat{p}_1)/15}$.

T F [03] e. The statement that $\Phi(1.96) = 0.975$ means that $P(Z > 1.96) = 0.975$, where Z is a standard normal random variable.