STA 580 — Spring 2011 — Dr. Charnigo

Written Assignment 5 Solutions

1a. We have $\bar{y}_1 = 90.50$, $s_1^2 = 18.86$, $\bar{y}_2 = 93.75$, $s_2^2 = 19.93$, $\bar{y}_3 = 94.25$, $s_3^2 = 14.50$, and $\bar{\bar{y}} = 92.833333$. Hence, Between SS is

$$8 \times (90.50^2 + 93.75^2 + 94.25^2) - 24 \times (92.833333)^2 = 66.33.$$

Also, Within SS is

$$(8-1) \times (18.86 + 19.93 + 14.50) = 373.0.$$

[Quoting SAS output on this and subsequent items is permissible.]

1b. Between MS is $66.33/(3-1) = 33.17$ and Within MS is $373.0/(24-3) = 17.76$, so $f = 33.17/17.76 = 1.87 < 3.47 = f_{2,21,0.95}$. [Since $f_{2,21,0.95}$ does not appear in Table 9 of your textbook, an approximation such as $f_{2,20,0.95} = 3.49$ is satisfactory for the purpose of this exercise. Alternatively, one may note that the SAS-reported p-value of 0.1793 is greater than 0.05.] Thus, we do not reject H_0 in favor of the complementary alternative.

1c. To test $H_0 : \mu_1 = \mu_2$ we calculate

$$t = \frac{90.50 - 93.75}{\sqrt{17.76(1/8 + 1/8)}} = -1.54.$$

Since $t_{21,0.975} = 2.08$, we are unable to reject $H_0 : \mu_1 = \mu_2$. [The SAS-reported p-value is 0.1379.]

To test $H_0 : \mu_1 = \mu_3$ we calculate

$$t = \frac{90.50 - 94.25}{\sqrt{17.76(1/8 + 1/8)}} = -1.78.$$

Since $t_{21,0.975} = 2.08$, we are unable to reject $H_0 : \mu_1 = \mu_3$. [The SAS-reported p-value is 0.0896.]

1d. The t statistics from 1c are unaltered, but now the original critical value of $t_{21,0.975} = 2.08$ must be replaced by $t_{21,0.9875} = 2.41$. [Since $t_{21,0.9875}$ does not appear in Table 5 of your textbook, an approximation such as $t_{21,0.99} = 2.52$ is satisfactory for the purpose of this exercise. Notice, however, that approximating via the nearest column in Table 5 is typically not as reliable as approximating via the nearest row.] Our decisions not to reject $H_0 : \mu_1 = \mu_2$ and not to reject $H_0 : \mu_1 = \mu_3$ are unaltered. [The SAS-reported p-values from part c could be doubled and then compared to 0.05.]

1e. Since $\mu_1 - 0.5\mu_2 - 0.5\mu_3 = 0$ is the same as $\mu_1 = (\mu_2 + \mu_3)/2$, this null hypothesis says that mean compliance at fast food restaurants is the same as the average of mean compliance at casual dining restaurants and mean compliance at fine dining restuarants. [The latter is an approximation to mean compliance at non fast food restaurants, which include both casual dining restaurants and fine dining restaurants.]

1f. We calculate

$$t = \frac{90.50 - 0.5 \times 93.75 - 0.5 \times 94.25}{\sqrt{17.76(1/8 + 0.25/8 + 0.25/8)}} = \frac{-3.50}{1.82} = -1.92.$$

With a Scheffe adjustment, the critical value is not $t_{21,0.975} = 2.08$ but rather $\sqrt{(3-1)f_{2,21,0.95}} = 2.63$. Hence, we are unable to reject H_0. [Since $f_{2,21,0.95}$ does not appear in Table 9 of your textbook, an approximate critical value of $\sqrt{(3-1)f_{2,20,0.95}} = 2.64$ is satisfactory for the purpose of this exercise.]

1g. We have $n_1 = n_2 = n_3 = 8$, $N = 24$, $r_1 = 71.5$, $r_2 = 110.0$, and $r_3 = 118.5$. Since there are six two-way ties and two three-way ties, the test statistic is

$$\frac{\frac{12}{24(25)} \times (71.5^2/8 + 110.0^2/8 + 118.5^2/8) - 3(25)}{1 - \frac{6(2^3-2)+2(3^3-3)}{24^3-24}} = 3.16.$$

Since the critical value is $\chi^2_{2,0.95} = 5.99$, we do not reject H_0. [The SAS-reported p-value is 0.2064.]

2a. We have $a = 3$, $b = 2$, balanced data with $n = 4$, $\bar{y}_{11} = 88.25$, $\bar{y}_{12} = 92.75$, $\bar{y}_{21} = 92.25$, $\bar{y}_{22} = 95.25$, $\bar{y}_{31} = 93.00$, $\bar{y}_{32} = 95.50$, $\bar{y}_{1.} = 90.50$, $\bar{y}_{2.} = 93.75$, $\bar{y}_{3.} = 94.25$, $\bar{y}_{.1} = 91.166667$, $\bar{y}_{.2} = 94.50$, and $\bar{y}_{..} = 92.833333$. Also, $s^2_{11} = 12.25$, $s^2_{12} = 18.25$, $s^2_{21} = 18.92$, $s^2_{22} = 21.58$, $s^2_{31} = 10.00$, and $s^2_{32} = 19.67$. Thus, we have

$$SST = 3(12.25+18.25+18.92+21.58+10.00+19.67)+4(88.25^2+92.75^2+92.25^2+95.25^2+93.00^2+95.50^2)$$
$$-24(92.833333)^2 = 439.33,$$
$$SSA = 8(90.50^2 + 93.75^2 + 94.25^2) - 24(92.833333)^2 = 66.33,$$
$$SSB = 12(91.166667^2 + 94.50^2) - 24(92.833333)^2 = 66.67,$$
$$SSAB = 4(88.25^2 + 92.75^2 + 92.25^2 + 95.25^2 + 93.00^2 + 95.50^2) - 24(92.833333)^2 - 66.33 - 66.67 = 4.33,$$

and

$$SSE = 439.33 - 66.33 - 66.67 - 4.33 = 302.00.$$

2b. In testing for nonzero interaction effects we have

$$f_{AB} = \frac{4.33/2}{302.00/18} = 0.13.$$

Since this does not exceed $f_{2,18,0.95} = 3.55$, we do not reject the null hypothesis of zero interaction effects. [The SAS-reported p-value is 0.8797.]

In testing for nonzero main effects from restaurant type we have

$$f_A = \frac{66.33/2}{302.00/18} = 1.98.$$

Since this does not exceed $f_{2,18,0.95} = 3.55$, we do not reject the null hypothesis of zero main effects from restaurant type. [The SAS-reported p-value is 0.1675.]

In testing for nonzero main effects from managerial staff training we have

$$f_B = \frac{66.67/1}{302.00/18} = 3.97.$$

Since this does not exceed $f_{1,18,0.95} = 4.41$, we do not reject the null hypothesis of zero main effects from managerial staff training. [The SAS-reported p-value is 0.0616.]