STA 623 — Fall 2009 — Dr. Charnigo

Midterm Examination

This non-collaborative take-home midterm examination may be submitted any time between 12 Noon on Monday 05 October and 2 p.m. on Thursday 08 October. By non-collaborative I mean that you are not permitted to discuss the examination with anyone other than me, until after the deadline for submission. The examination is to be submitted in hard copy, to me in person or under my office door.

[50] 1. Let X have probability density function $f_X(x) := C/(1 + [x/\sigma]^2)$, where σ is a positive real.

[10] a. What is C?

[15] b. Let $g(x) := 1/x$ for $x \neq 0$ and $g(0) := 0$. Put $Y := g(X)$. What is the cumulative distribution function of Y?

[10] c. Find a probability density function for Y.

[15] d. Find $P(X > \sigma^2 Y > 0)$. [Hint: Express $\{X > \sigma^2 Y > 0\}$ as $\{X \in \mathcal{I}\}$ for some interval $\mathcal{I} \subset \mathbb{R}$.]

[25] 2. Let X have probability mass function $f_X(x) := \exp[-\lambda]\lambda^x/x!$ for $x \in \{0, 1, 2, \ldots\}$, where λ is a positive real.

[10] a. Find $E[X(X-1)]$ by explicitly evaluating the infinite sum $\sum_{x=0}^{\infty} x(x-1)\exp[-\lambda]\lambda^x/x!$. Use your answer to calculate $E[X^2]$.

[15] b. Prove that the moment generating function is given by $M_X(t) := \exp[\lambda(\exp[t] - 1)]$. Use this result to verify your calculation of $E[X^2]$ from part a. What is $Var[X]$?

[25] 3. Let X have probability density function $f_X(x) := \frac{\Gamma[\alpha+\beta]}{\Gamma[\alpha]\Gamma[\beta]} x^{\alpha-1}(1-x)^{\beta-1}$ for $x \in (0, 1)$, where α and β are positive reals.

[10] a. Prove that $E[\exp(tX)]$ exists as a finite number for any real t. [Do not attempt to calculate the expected value, just show that it is a finite number.]

[15] b. Find $E[X^\gamma(1-X)^\delta]$, where γ and δ are nonnegative reals.