STA 623 — Fall 2009 — Dr. Charnigo

Written Assignment 1

Written Assignment 1 is due on Thursday 10 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer. In what follows, all sets are assumed to be events that belong to the sigma field.

[10] 1. Give a careful proof of the second DeMorgan law, $(A \cap B)^c = A^c \cup B^c$.

[10] 2. Give a careful proof of the assertion that $\cup_{n=1}^{\infty} \cap_{i=n}^{\infty} A_i = B$, where B is defined to be the set containing each element of S present in all but finitely many of $A_1, A_2, A_3, \ldots$.

[10] 3. A sports writer states that the odds against Horse A winning the Kentucky Derby are 2:1 (i.e., the probability of Horse A not winning is twice the probability of Horse A winning), the odds against Horse B winning the Kentucky Derby are 3:1, the odds against Horse C winning the Kentucky Derby are 4:1, the odds against Horse D winning the Kentucky Derby are 5:1, the odds against Horse E winning the Kentucky Derby are 6:1, and the odds against Horse F winning the Kentucky Derby are 7:1. Assuming that only one Horse can win the Kentucky Derby, determine whether the sports writer's statement is compatible with the axioms of probability.

[10] 4. Show that

$$P(A_1) + P(A_2) - 1 \leq P(A_1 \cap A_2) \leq \min\{P(A_1), P(A_2)\}.$$

Then show that

$$\sum_{i=1}^{n} P(A_i) - n + 1 \leq P(\cap_{i=1}^{n} A_i) \leq \min\{P(A_1), \ldots, P(A_n)\}.$$

[10] 5. During the next three days (let's say Monday, Tuesday, and Wednesday) I will receive two phone calls, the calls being randomly distributed among the three days. Determine the probability that I will receive at least one phone call on Monday. Then find the flaw in the following argument: "This is like a problem in which I am selecting r objects from n objects, with $r = 2$ and $n = 3$. The objects can be replaced (I can receive both phone calls on the same day), and the order in which the objects are selected does not matter (getting one call on a Monday and another call on a Tuesday is the same as getting one call on a Tuesday and another call on a Monday). Thus there are $\binom{4}{2} = 6$ possibilities, which in fact I can list: both calls on Monday, both calls on Tuesday, both calls on Wednesday, one call on Monday and another Tuesday, one call on Tuesday and another Wednesday, one call on Monday and another Wednesday. I note that Monday appears in three of these possibilities. Therefore the probability of receiving at least one phone call on Monday is 3/6."

[10] 6. I will be dealt a 10-card gin hand from a well-shuffled standard 52-card deck. What is the probability that I will receive two triples and two pairs?

[10] 7. I bought a lottery ticket with the numbers $7, 11, 12, 18, 35, 42$. I will win a large prize if all six of these numbers appear on the six balls drawn without replacement from a vat containing 44 balls, but I will win a small prize if four or five of these numbers appear. What is the probability that I will win a prize?

[10] 8. Suppose that 0.6% (6/10 of 1%, not 60%) of people in the U.S. have HIV and that a diagnostic test is available such that: (i) 95% of people who really have HIV test positive; and, (ii) 95% of people who really do not have HIV test negative. If a person tests positive for HIV, what is the probability that this person actually has HIV?

[10] 9. Suppose that I roll a six-sided die. Let $A := \{2, 4, 6\}$ and $B := \{3, 6\}$. If

$$P(\{1\}) = P(\{2\}) = P(\{3\}) = P(\{4\}) = P(\{5\}) = P(\{6\}) = 1/6,$$

then show that A and B are independent. If

$$P(\{1\}) = P(\{2\}) = P(\{3\}) = 1/9 \quad \text{and} \quad P(\{4\}) = P(\{5\}) = P(\{6\}) = 2/9,$$

then show that A and B are not independent. Can A and B be independent without

$$P(\{1\}) = P(\{2\}) = P(\{3\}) = P(\{4\}) = P(\{5\}) = P(\{6\}) = 1/6?$$

[10] 10. Under what conditions, if any, are two events A, B both independent and mutually exclusive?