STA 623 — Fall 2009 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Thursday 24 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[60] 1. Suppose that we conduct a taste test in which each of 3 shoppers is asked to try "Brand A" and "Brand B" of peanut butter and then state which one he or she prefers. Assume that different shoppers respond independently and that the probability of any particular shopper preferring "Brand A" is some number $p \in [0, 1]$. Let E_i denote the event that shopper i prefers "Brand A" for $i \in \{1, 2, 3\}$, and let X be the total number of shoppers who prefer "Brand A".

[05] a. Express $\{X = 0\}$ in terms of E_1, E_2, E_3 and determine $P(X = 0)$ accordingly.

[05] b. Express $\{X = 1\}$ in terms of E_1, E_2, E_3 and determine $P(X = 1)$ accordingly.

[05] c. Express $\{X = 2\}$ in terms of E_1, E_2, E_3 and determine $P(X = 2)$ accordingly.

[05] d. Express $\{X = 3\}$ in terms of E_1, E_2, E_3 and determine $P(X = 3)$ accordingly.

[10] e. What is the cumulative distribution function of X?

[10] f. Let Y denote the total number of shoppers who prefer "Brand B". Clearly, X and Y are not the same. However, are there any conditions under which X and Y are identically distributed?

[10] g. Now suppose that the taste test is expanded to include n shoppers, where n is some positive integer much larger than 3. What is the probability mass function of X?

[10] h. Find a formula for $\widehat{p} := \arg\max_{p \in [0,1]} P(X = x)$ as a function of $x \in \{0, 1, \ldots, n\}$. If we did not know the value of p a priori and wanted to guess it after observing that x shoppers preferred "Brand A", one possible guess would be $\widehat{p}$. Such a guess would be called a "maximum likelihood estimate" of p.

[40] 2. Suppose that X has probability density function $Cx(1 - x)1_{\{x \in (0,1)\}}$ for some constant C.

[10] a. What is C?

[10] b. What is the cumulative distribution function of X?

[10] c. What is the cumulative distribution function of $Y := -\log X$?

[10] d. What is a probability density function of Y?