STA 623 — Fall 2009 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Thursday 22 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[20] 1. Let X denote the number of Bernoulli trials required to achieve one success *and* one failure.

[10] a. Discuss the choice of parameter space for p.

[10] b. Noting that the outcome of the last trial must oppose the outcome of all preceding trials, find the probability mass function of X.

Remark: One way to check your answer is to see whether $\sum_{x \in \mathcal{X}} P(X = x) = 1$. If you do not have $\sum_{x \in \mathcal{X}} P(X = x) = 1$, then your answer is not correct.

[20] 2. Let X have the offset geometric distribution with parameter $p \in (0, 1]$. For convenience put $q := 1 - p$.

[10] a. Starting from $\sum_{x=1}^{\infty} xq^{x-1} = \frac{1}{(1-q)^2}$, justify the interchange of differentiation and summation to show that $\sum_{x=2}^{\infty} x(x-1)q^{x-2} = \frac{2}{(1-q)^3}$ for $q \in (0, 1)$. Then show that $\sum_{x=2}^{\infty} x(x-1)q^{x-2} = \frac{2}{(1-q)^3}$ for $q = 0$ simply by calculating both sides.

[10] b. Use the results from part a to find $E[X]$ and $Var[X]$.

[20] 3. Suppose that, for any fixed $t \in [0, \infty)$, the number of phone messages arriving before time t has the Poisson distribution with mean λt for some $\lambda \in (0, \infty)$. Moreover, suppose that the number of text messages arriving before time t has the Poisson distribution with mean μt for some $\mu \in (0, \infty)$. Finally, suppose that phone messages arrive independently of text messages.

[10] a. Let T denote the time at which the first phone message is received. Find an expression for $P(T > t)$. What is the distribution of T?

[10] b. Let U denote the time at which the first text message is received, and let $V := \min\{T, U\}$ denote the time at which the first message of either kind is received. Derive an expression for $P(V > t)$ by using relationships among $\{T > t\}$, $\{U > t\}$, and $\{V > t\}$. What is the distribution of V?

Remark: One way to check your answer involves noting that the total number of messages arriving before time t has the Poisson distribution with mean $(\lambda + \mu)t$.

[20] 4. Let X_n have the Poisson distribution with mean $\lambda_n := n$ for $n \in \{1, 2, \ldots\}$. Put $Z_n := n^{-1/2}(X_n - n)$.

[10] a. Given that $M_{X_n}(t) = \exp[n(\exp[t] - 1)]$, derive a formula for $M_{Z_n}(t)$.

[10] b. To what does $M_{Z_n}(t)$ converge as $n \to \infty$? What are the implications for approximating Z_n (and hence X_n) as n becomes large?

[20] 5. Let X have the probability density function $f(x) := 1_{\{x \in (0,\infty)\}} \beta/(x + 1)^{\beta+1}$ for some $\beta \in (0, \infty)$.

[10] a. Compute the survival and hazard functions for X.

[10] b. Identify one physical, social, or biological phenomenon for which X appears to be an unsuitable probabilistic model. Identify another phenomenon for which X may be an appropriate probabilistic model.