STA 623 — Fall 2010 — Dr. Charnigo

Final Examination

This take-home non-collaborative final examination, worth 30% of your grade, originally had a due date of Tuesday 14 December at 2 p.m. (By non-collaborative I mean that you are not permitted to discuss the examination with anyone other than me, until after the deadline for submission.) Because I will be attending a conference during finals week, I am hereby extending the due date to Wednesday 15 December at 2 p.m., with the stipulation that there will not be a corresponding extension of the customary 24-hour grace period for 75% credit. In other words, a final examination submitted on Thursday 16 December will receive no credit whatsoever. In addition, I need you to submit the final examination in one of the following three ways:

a. To me in person or under my office door (203-B College of Public Health) anytime before 5:30 p.m. on Monday 13 December.

b. To me via e-mail anytime before 2 p.m. on Wednesday 15 December, in either PDF or DOC format. (If you prepare your solutions using pencil and paper, you can employ a scanner to make a PDF file.) Also carbon copy the e-mail to both the Biostatistics Department administrative assistant {johanna.startzman@uky.edu} and yourself.

c. To the Biostatistics Department administrative assistant, Johanna Startzman, in person anytime before 2 p.m. on Wednesday 15 December. You may need to e-mail her a day or two in advance to arrange a drop-off time since she has two offices on campus and moves between both of them.

[50] 1. Suppose that X has the chi-square distribution on k (positive integer) df and that Y, independent of X, has the chi-square distribution on 1 df.

[20] a. Put $U := X + Y$ and $V := X$. Find the joint probability density function of U and V. Are U and V independent? Did you expect them to be?

[15] b. Find the marginal probability density function of U by integrating the joint probability density function of U and V in dv.

Hint. You may quote without proof that, by trigonometric substitution $\sin^2\theta := v/u$,

$$\int_0^u v^{k/2-1}/\sqrt{u-v}\ dv = 2u^{(k-1)/2}\int_0^{\pi/2}\sin^{k-1}(\theta)\ d\theta = u^{(k-1)/2}\Gamma[k/2]\Gamma[1/2]/\Gamma[(k+1)/2].$$

[15] c. As a check of your answer to part b, use moment generating functions to find the

distribution of $X+Y$. You may quote without proof the well-known result that a gamma random variable with shape $\alpha(>0)$ and scale $\beta(>0)$ has moment generating function $(1-\beta t)^{-\alpha}$ for $t<1/\beta$.

[50] 2. Let X have probability density function $f(x) := 1_{\{0<x<1\}}$.

[15] a. Find the probability density function of $Y := -c^{-1}\log X$, where c is a positive constant.

[15] b. Use your answer to part a along with the kernel method to find the expected value of $Y^a \exp(-bY)$, where a and b are positive constants.

[20] c. Find the probability density function of $W := z_X$, where z_x denotes the x quantile of the standard normal distribution for $0<x<1$.

Hint. Letting Φ denote the cumulative distribution function of the standard normal distribution, we have $\Phi(z_x) = x$. As such, we have $g^{-1}(y) = \Phi(y)$ when $g(x) := z_x$.

Remark. Since this item can be handled similarly if z_X is replaced by $t_{k,X}$ or $\chi^2_{k,X}$, etc., the practical implication of this item is that you can simulate an observation from any continuous distribution of practical interest by first simulating an observation from the uniform distribution on $(0,1)$ and then applying an appropriate transformation.