STA 623 — Fall 2010 — Dr. Charnigo

Midterm Examination

This non-collaborative take-home midterm examination is due at the end of class on Thursday 07 October. By non-collaborative I mean that you are not permitted to discuss the examination with anyone other than me, until after the deadline for submission. The examination is to be submitted in hard copy, to me in person or under my office door.

[50] 1. Let X have probability mass function $f_X(x) := \binom{n}{x} p^x (1-p)^{n-x}$ for $x \in \{0, 1, 2, \ldots, n\}$, where $n \in \{1, 2, \ldots\}$ and $p \in (0, 1)$.

[10] a. Show that $E[X] = np$ by evaluating the sum $\sum_{x=0}^{n} x f_X(x)$.

Hint: Begin by proving that $x\binom{n}{x} = n\binom{n-1}{x-1}$ for $x \in \{1, 2, \ldots, n\}$.

[15] b. Show that $E[X(X-1)] = n(n-1)p^2$ by evaluating the sum $\sum_{x=0}^{n} x(x-1) f_X(x)$. Use this and part a to find $E[X^2]$ and $Var[X]$.

[10] c. Show that the moment generating function is $M_X(t) = [pe^t + (1-p)]^n$.

Hint: You may use without proof the well-known fact that $(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x}$ for any real numbers a and b, where $n \in \{1, 2, \ldots, \}$.

[15] d. Use part c to find $E[X]$, providing a check on part a.

[25] 2. Let X have probability density function $f_X(x) := (2\pi)^{-1/2} \exp[-x^2/2]$.

[10] a. Find the probability density function of $Y := \exp[X]$.

[15] b. Find the probability density function of $Z := X^2$.

Hint: Show that, for $z \geq 0$, we have $P(Z \leq z) = 2\int_0^{\sqrt{z}} f_X(x)dx$. Then employ the Leibniz rule for differentiating an integral with a variable upper limit, {http://www.math.carleton.ca/∼amingare/calculus/leibniz.html}.

[25] 3. The mayor of a small town wants to appoint a five-person task force to make a recommendation for implementation of a smoking ban. The mayor's "short list" includes four members of city council, three employees of the public health department, and three faculty members at the University.

[10] a. How many ways are there to appoint the task force so that at least one faculty member is included?

Hint: First note how many ways there are to appoint the task force without restriction. Then find how many ways there are to appoint the task force so that no faculty member is included.

[15] b. How many ways are there to appoint the task force so that at least one faculty member and at least one public health employee are included?