STA 623 — Fall 2010 — Dr. Charnigo

Written Assignment 1

Written Assignment 1 is due on Thursday 09 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer. In what follows, all sets are assumed to be events that belong to the sigma field.

[10] 1. Give a careful proof of the first distributive law, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

[10] 2. Prove result 5 that $P(A) \leq 1$ without using any subsequent result. (Thus, you are limited to result 4 and axioms 1, 2, and 3.)

[10] 3. Derive a general formula for $P(A \cup B \cup C)$ in terms of probabilities involving A, B, C, and their intersections. (As a first step, put $D := B \cup C$.)

[10] 4. Draw a Venn diagram that provides an intuitive visual interpretation of the formula derived in exercise 3.

[10] 5. I will be dealt 10 cards from a well-shuffled standard 52-card deck. What is the probability that I will receive two triples, one pair, and two singles?

[10] 6. I just paid a dollar for a lottery ticket with the numbers 1, 6, 11, 16, and 21. I will win a large prize if all five of these numbers appear on the five balls drawn without replacement from a vat containing 40 balls, but I will win a small prize if four of these numbers appear. What is the probability that I just wasted a dollar?

[20] 7. Suppose that $S = \{1, 2, 3, 4, 5, 6\}$, as if I were rolling a six-sided die. Show that any sigma field containing $\{2, 4, 6\}$ and $\{3, 6\}$ must also contain $\{1, 5\}$,$\{2, 4\}$,$\{3\}$, and $\{6\}$. Also show that $\{2, 4, 6\}$ and $\{3, 6\}$ must be contained in any sigma field containing $\{1, 5\}$,$\{2, 4\}$,$\{3\}$, and $\{6\}$. Conclude that the smallest sigma field containing $\{2, 4, 6\}$ and $\{3, 6\}$ consists of $\emptyset$, $\{1, 5\}$,$\{2, 4\}$,$\{3\}$, $\{6\}$, and their unions.

[20] 8. Suppose that 0.7% (7/10 of 1%, not 70%) of people in the U.S. have HIV and that a diagnostic test is available such that: (i) 90% of people who really have HIV test positive; and, (ii) P% of people who really do not have HIV test negative, where P is some number between 0 and 100. How large must P be so that half of people testing positive for HIV actually have HIV?