STA 623 — Fall 2010 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Thursday 23 September at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[40] 1. This is a continuation of exercise 7 from Written Assignment 1. Let $S := \{1, 2, 3, 4, 5, 6\}$ be endowed with the sigma field consisting of $\emptyset$, $\{1, 5\}$, $\{2, 4\}$, $\{3\}$, $\{6\}$, and their unions. Suppose, moreover, that $P(\{1, 5\}) = 1/3$, $P(\{2, 4\}) = 1/6$, $P(\{3\}) = 1/3$, and $P(\{6\}) = 1/6$. Note, first, that this is not a fair die and, second, that I have not attempted to assign a probability to $\{1\}$, $\{2\}$, $\{4\}$, or $\{5\}$ individually.

[10] a. Let X denote the outcome of rolling the die. That is, we have $X(\omega) := \omega$ for any $\omega \in S$. Show that X is not a random variable.

Hint: Exhibit a set $B \in \mathcal{B}^1$ such that $X^{-1}(B) := \{\omega : X(\omega) \in B\}$ does not belong to the sigma field on S.

[10] b. Suppose I win a dollar if the die comes up even and I lose a dollar if the die comes up odd. Let Y denote my winnings. That is, we have $Y(\omega) := 1 - 2(\omega \bmod 2)$. Show that Y is a random variable.

Hint: At first, you may think I am asking the impossible because there are infinitely many ways to choose $B \in \mathcal{B}^1$ and we must have $Y^{-1}(B) := \{\omega : Y(\omega) \in B\}$ belong to the sigma field on S for each of them. Unfortunately, you can't do infinitely many computations in finite time! However, there are really only four cases that require consideration:

(i) B contains neither -1 nor $+1$;

(ii) B contains -1 but not $+1$;

(iii) B contains $+1$ but not -1; and,

(iv) B contains both -1 and $+1$.

So, for each of these four cases, find $Y^{-1}(B)$ and show that it belongs to the sigma field on S.

[10] c. What is the probability mass function of Y?

[10] d. What is the cumulative distribution function of Y?

[10] 2. For what values of C does $[1 - C/(1 + x^2)]1_{\{x \geq 0\}}$ define a valid cumulative distribution function? Please give a reason for your answer.

[10] 3. For what values of C does $[1 - C/(1 + x^2)]1_{\{x > 0\}}$ define a valid cumulative distribution function? Please give a reason for your answer.

[40] 4. Suppose that the random variable X has cumulative distribution function as in exercise 3.

[10] a. Find the probability density function of X.

[10] b. Find the probability density function of $Y := \sqrt{1 + X^2}$.

[10] c. Find the cumulative distribution function of $Z := |X - 2|$.

Hint: In evaluating $P(Z \leq z)$, consider three cases: $z \in (-\infty, 0)$, $z \in [0, 2]$, and $z \in (2, \infty)$.

[10] d. Find the probability density function of Z.