STA 623 — Fall 2010 — Dr. Charnigo

Written Assignment 3

Written Assignment 3 is due on Thursday 21 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[50] 1. Three contestants on the Price is Right television game show spin a big wheel, which is numbered from 5 to 100 in increments of 5. If a contestant spins anything less than 100, he/she is given the option to spin a second time and count the sum of the two spins as his/her score. However, a contestant whose score exceeds 100 is disqualified. The non-disqualified contestant with the highest score wins the right to compete in the Showcase Showdown. (If necessary, a tiebreaker round can be performed.)

Suppose that the result of any given spin is a discrete uniform random variable with support set $\{5, 10, 15, \ldots, 95, 100\}$, that the results of different spins are independent, and that any contestants participating in the tiebreaker have the same probability of winning. (These are simplifications of reality because, among other things, contestants know the position of the 100 on the wheel and so try to calibrate accordingly how hard they spin the wheel.)

[15] a. Suppose that the first contestant is disqualified and that the second contestant scores 85. What is the probability that the third contestant wins? *Hint*: If the third contestant gets anything less than 85 on his/her first spin, he/she will spin a second time. If the third contestant gets anything more than 85 on his/her first spin, he/she will not spin a second time. If the third contestant gets 85 exactly, he/she will not spin a second time since the probability of winning via the tiebreaker is greater than the probability of winning via a second spin.

[15] b. Repeat part a if the second contestant scores 90, 95, or 100.

[20] c. Suppose that the first contestant is disqualified and that you, the second contestant, get 85 on your first spin. You do not know what the third contestant will score. Which is your better strategy: to spin again or to stay with 85?

[30] 2. This exercise is a continuation of exercise 3 from Written Assignment 3 in Fall 2009. You may use without proof the Solutions to Written Assignment 3 in Fall 2009.

[10] a. Let $W := \max\{T, U\}$ denote the time by which at least one message of each kind is received. Find the cumulative distribution function of W.

[10] b. What is the expected value of W? *Hint*: Review page 4 of the Section 2.2 notes.

[10] c. What is the probability density function of $\exp[-W]$?

[20] 3. For $n \in \{1, 2, \ldots\}$, let X_n have the negative binomial distribution with parameters $p_n := \max\{0, 1 - \lambda/n\}$ and $r_n := n$, where λ is a fixed positive real number. In what follows, you may use without proof the moment generating functions recorded on page 622 of your textbook.

[10] a. Evaluate $\lim_{n\to\infty} M_{X_n}(t)$. *Hint*: Note that $1 - \lambda/n = 1 - \exp[t]\lambda/n + (\exp[t] - 1)\lambda/n$.

[10] b. Evaluate $\lim_{n\to\infty} P(X_n \leq x)$ for x not a nonnegative integer.