STA 623 — Fall 2010 — Dr. Charnigo

Written Assignment 4

Written Assignment 4 is due on Thursday 04 November at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[60] 1. In this exercise, you may use without proof the series expansions $-\log[1+q] = \sum_{x=1}^{\infty}(-q)^x/x$ and $-q/[1+q] = \sum_{x=1}^{\infty}(-q)^x$, valid for $|q| < 1$.

Put $f(x;p) := -(1-p)^x/(x \log p)$ for $x \in \{1, 2, \ldots\}$ and $p \in (0,1)$.

[10] a. Show that $f(x;p)$ defines a valid probability mass function.

[10] b. Calculate $E[X]$ directly.

[10] c. Calculate $E[X^2]$ directly and use it to find $Var[X]$. (Formula (1) from the Section 2.4 notes, which was justified in those same notes, may be helpful.)

[10] d. Show that $f(x;p)$ belongs to an exponential family.

[20] e. Use the moment calculation formulas for exponential families to verify your answers to parts b and c.

[20] 2. Prove that the family of uniform distributions $\{1_{\{a<x<b\}}/(b-a) : -\infty < a < b < \infty\}$ is a location-scale family by exhibiting a probability density function $f(x)$ such that $\{1_{\{a<x<b\}}/(b-a) : -\infty < a < b < \infty\} = \{\sigma^{-1} f((x-\mu)\sigma^{-1}) : \mu \in (-\infty, \infty), \sigma \in (0, \infty)\}$ and relating μ, σ to a, b.

[20] 3. In this exercise, you may use without proof the fact that $\int_0^{\infty} x^{1/3}/(1+x^2)\ dx = \pi/\sqrt{3}$. (This formula can be derived using the residue theorem from complex analysis.)

Let X have the Cauchy distribution with location parameter 0 and scale parameter 1. Let δ be a positive real number. Establish a finite upper bound for $P(|X| > \delta)$ by using a version of Chebychev's Inequality with $g(X) := |X|^{1/3}$.