STA 623 — Fall 2010 — Dr. Charnigo

Written Assignment 5

Written Assignment 5 is due on Thursday 18 November at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

[30] 1. Use moment generating functions to find the distribution of $X+Y$, X and Y independent, in each of the following situations. You may take for granted that the moment generating functions are as reported in the back of your textbook; you do not need to derive them from scratch.

[10] a. X negative binomial with parameters $r_1 \in \{1, 2, \ldots\}$ and $p \in (0, 1]$, Y negative binomial with parameters $r_2 \in \{1, 2, \ldots\}$ and $p \in (0, 1]$

[10] b. X Poisson with parameter $\lambda_1 \in (0, \infty)$, Y Poisson with parameter $\lambda_2 \in (0, \infty)$

[10] c. X normal with parameters $\mu_1 \in (-\infty, \infty)$ and $\sigma_1 \in (0, \infty)$, Y normal with parameters $\mu_2 \in (-\infty, \infty)$ and $\sigma_2 \in (0, \infty)$.

[70] 2. Let X and Y have joint probability density function

$$f_{X,Y}(x, y) = \left(\frac{4}{\pi[\log 4 - 1]} \right) \log(1 + x^2 + y^2) 1_{\{x>0, y>0, x^2+y^2<1\}}.$$

[10] a. Create a visual representation of $f_{X,Y}(x, y)$. *Hint*: One option, but certainly not the only option, is to use the "persp" function in the R software package.

[10] b. Find the marginal probability density function of X. *Hint*: Try integrating by parts with $u := \log[1+x^2+y^2]$. Letting $h(x)$ be shorthand for $1+x^2$, note that $y^2/(h(x)+y^2) = 1-h(x)/(h(x)+y^2) = 1 - 1/(1 + [y/\sqrt{h(x)}]^2)$.

[10] c. Find the conditional probability density function of Y given that $X = x$ for $x \in (0, 1)$.

[10] d. Find the joint probability density function of $U := X^2$ and $V := Y^2$.

[10] e. Find the marginal probability density function of U. *Hint*: Instead of trying to integrate the joint probability density function of U and V in dv, use your answer to part b.

[10] f. Put $R := \sqrt{X^2 + Y^2}$ and $\Theta := \arctan(Y/X)$. Noting that $X = R\cos\Theta$ and $Y = R\sin\Theta$, find the joint probability density function of R and Θ.

[10] g. Use the visual representation in part a to give an intuitive explanation of why: (i) X and Y are not independent; but, (ii) R and Θ are independent.