STA 623 — Fall 2011 — Dr. Charnigo

Written Assignment 2

Written Assignment 2 is due on Tuesday 04 October at the end of class. You are encouraged to work in groups of two or three, but you may work individually if you prefer.

Put $f_X(x) := C(\log x)^{-\alpha}x^{-1}1_{\{x\geq e\}}$.

[10] 1. For what values of α can a value of C be chosen to make $f_X(x)$ a probability density function? Specify the value of C corresponding to each such value of α. In subsequent items, assume that $\alpha = 3$.

[10] 2. Show that $\lim_{x\to\infty}(\log x)^3 x^{-1} = 0$. Hence, there exists $K \in (0, \infty)$ such that $x^{-1} \leq (\log x)^{-3}$ whenever $x \geq K$.

[10] 3. Show that $\int_K^\infty (\log x)^{-3}\, dx = \infty$. Conclude that $E[X] = \infty$.

[10] 4. On the other hand, show that $E[\log X]$ is finite. What is its value?

[10] 5. Put $Y := \log X$. Find $f_Y(y)$, the probability density function of Y.

[10] 6. Directly evaluate $\int_1^\infty y f_Y(y)\, dy$. Notice that you get the same answer as in part d. Why? (The fact that you can calculate this quantity without knowing $f_Y(y)$ is sometimes called the law of the unconscious statistician.)

[10] 7. Show that Y has infinite second moment.

[10] 8. Find the cumulative distribution function of Y.

[10] 9. Let $Z := \lfloor Y \rfloor$. Find the probability mass function of Z.

[10] 10. Prove that $E[Z]$ is finite. (You are not asked to supply its value.)